Perturbative area-metric gravity

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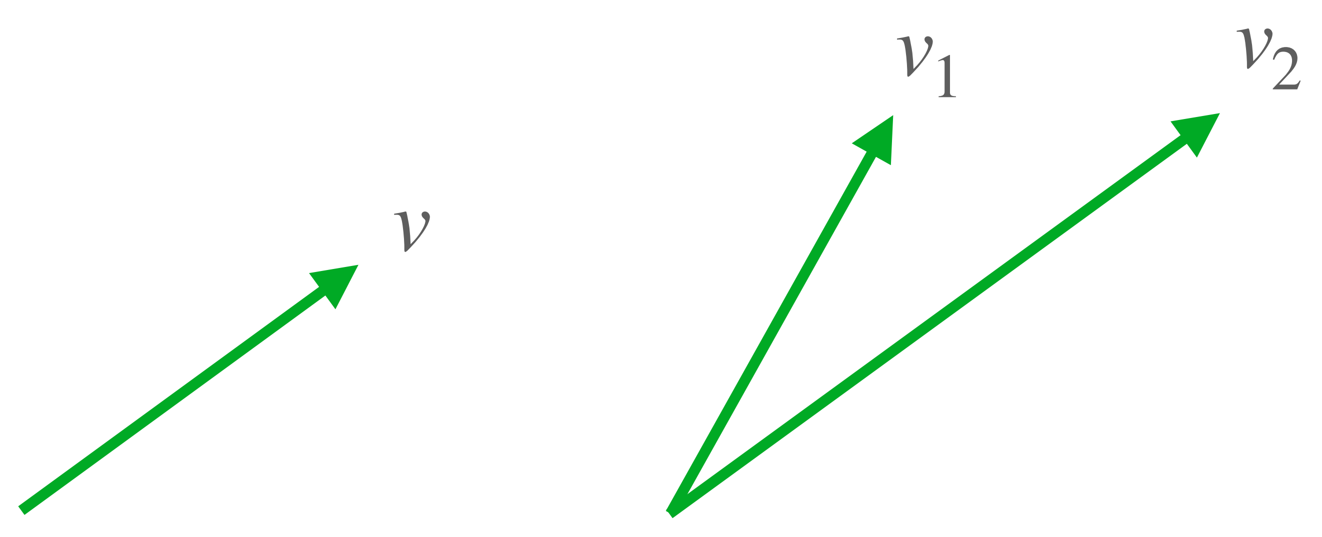
Area metric

Metric

$g_{\mu\nu} = g_{\nu\mu}$



$g(v,v) : l_v^2$ $\qquad$ $g(v_1, v_2) : \angle_{2d}(v_1, v_2)$

v : vector

g-induced area metric

$G_{\mu\nu\rho\sigma}(g) = g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\rho\nu}$

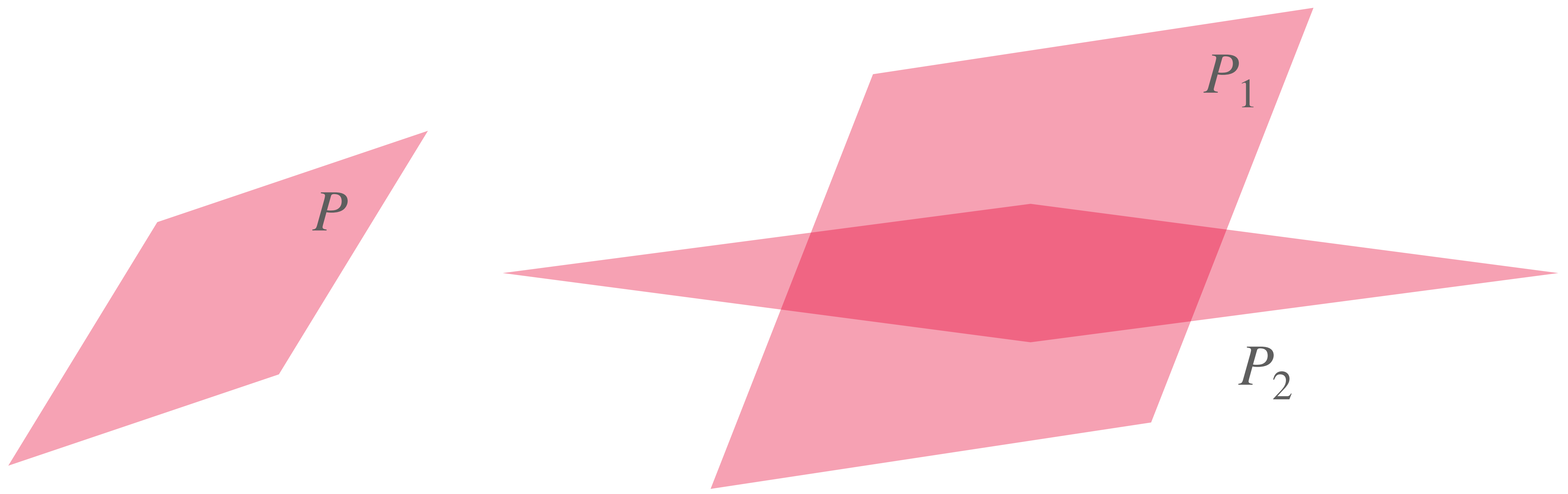
e.g.:
ED, YM: $S \supset G_{\mu\nu\rho\sigma}(g)F^{\mu\nu}F^{\rho\sigma}$, Nambu-Goto: $S = \mathrm{Area}(\Sigma) \supset G(g)$
$G(g) \mapsto G$ generalized background

Area metric

$G_{\mu\nu\rho\sigma} = G_{\rho\sigma\mu\nu} = -\,G_{\nu\mu\rho\sigma}$

$G_{\mu[\nu\rho\sigma]} = 0$

[Schuller & Wohlfahrt 2005, Punzi et al 2006]



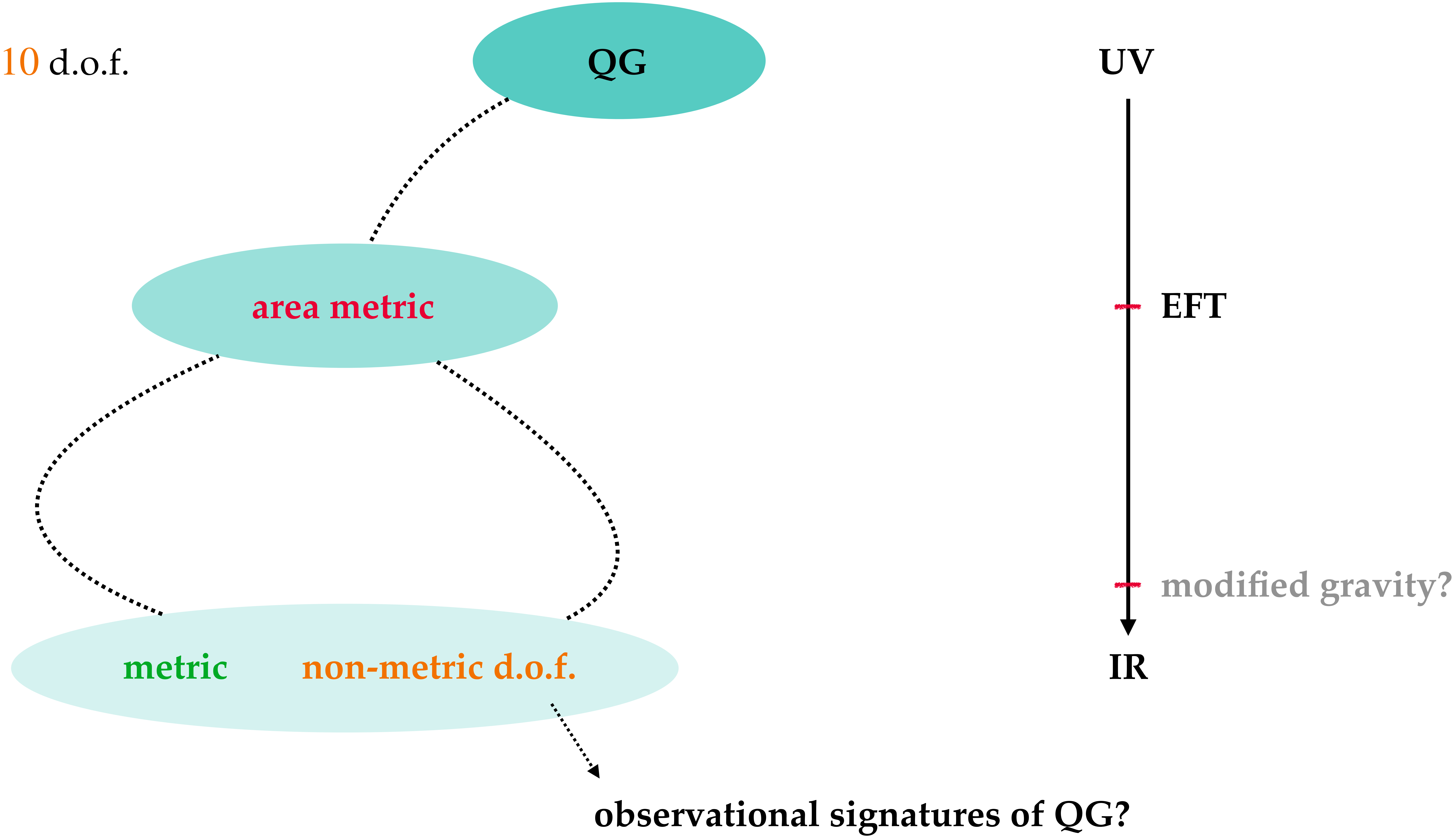
$G(P;P) : A_P^2$ $\qquad$ $G(P_1;P_2) : \angle_{3d}(P_1, P_2)$

P : (simple) bi-vector

$d = 4 :\ 21 - 1 =$ 20 d.o.f.

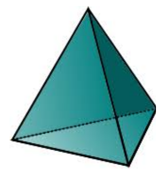
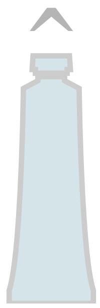
Possible scenario

$G : 20 = 10 + 10$ d.o.f.

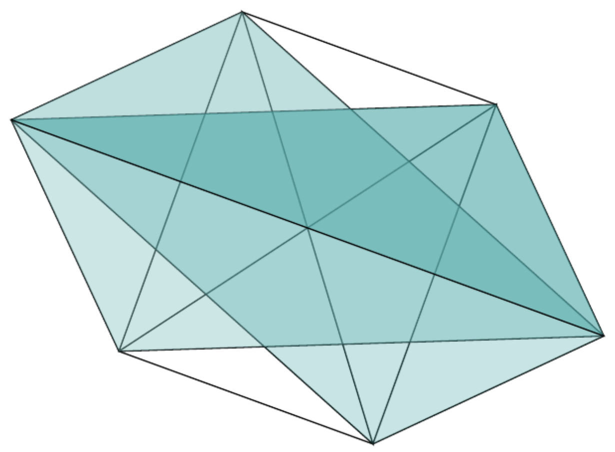


Semiclassical regime of effective spin foams

Twisted simplex geometry [Dittrich, Padua-Argüelles 2023]

semiclassical σ $= 5 \cdot$ classical  $+$  $\leftrightarrow$ $G(\sigma)$ microscopic

Lattice continuum limit of Area-Regge action [Dittrich 2021, Dittrich & Kogios 2022]



$$S[A_{t=\triangle}] = \sum_t A_t \epsilon(A_t) \;\to\; \lim_{\lambda \to 0} :\; S[G] \;\leftrightarrow\; G(g + \text{massive d.o.f.}) \text{ macroscopic}$$

$$\mathscr{L}^{(2)}_{eff}(h) = \mathscr{L}^{(2)}_{EH}(h) + {}^{(1)}\mathrm{Weyl}^2(h) + \mathcal{O}(\lambda^4)$$

glued simplices:

$n_{A_t} > n_{l_e}$

Continuum effective actions for spin foams

spin foams $\hat{=} \int_{\mathscr{C}} \mathscr{D}\mu e^{iS}$ where $\mathscr{C} > \{[g]\}$ due to 1st-class (sharp) + 2nd-class (weak) constraints

continuum analogue: modified non-chiral SO(4) Plebanski theories (subclass)

[cf. e.g.: Plebanski 1997, Pietri & Freidel 1999, Krasnov 2006-9, Freidel 2008, Speziale 2010, Beke et al 2012]

$$S = \int B^{IJ} \wedge F^{KL}(\omega) \text{ (topological)} + \frac{1}{2\gamma} \int \epsilon_{IJKL} B^{IJ} \wedge F^{KL}(\omega) \text{ (Holst)} - \frac{1}{2} \phi_{IJKL} B^{IJ} \wedge B^{KL} \text{ (20 simplicity constraints)}$$

20 constraints on $B^{IJ}_{\mu\nu}$ $\mapsto$ 10 constraints + 10 d.o.f. $V(\phi)$ $\Rightarrow$ non-linear $S[G]$ where $G \leftrightarrow g$ + 10 massive scalars $q^{\pm}$

[JB & Dittrich 2022]

Continuum effective actions for spin foams

[JB & Dittrich 2022]

perturbative inversion: $G = G(\delta) + a,\ g = \delta + h,\ q^{\pm} = \delta + \chi^{\pm}$ where $a \leftrightarrow h + \chi^{\pm}$

$$\mathscr{L}^{(2)}(a) = \mathscr{L}^{(2)}_{EH}(h) + \frac{1}{2}\sum_{\pm}\sqrt{\gamma_{\pm}}h_{\mu\nu}\chi^{\pm\mu\nu}p^2 + \frac{1}{2}(p^2 - m^2_{\pm})\chi^{\pm}_{\mu\nu}\chi^{\pm\mu\nu}, \quad \gamma_{\pm} = 1 \pm \frac{1}{\gamma}$$

effective length-metric action for $m^2_{\pm} \equiv m^2$**:**

$$\mathscr{L}^{(2)}_{eff}(h) = \mathscr{L}^{(2)}_{EH}(h) - {}^{(1)}C_{\mu\nu\rho\sigma}(h)\frac{1}{p^2 - m^2}{}^{(1)}C^{\mu\nu\rho\sigma}(h)$$

LQG area spectrum
[Rovelli & Smolin 1994, Ashtekar & Lewandowski 1996]
$A_j = \gamma l^2_{pl}\sqrt{j(j+1)}$
γ : Immirzi parameter = parity-violating coupling in area-metric gravity

Diffeomorphism-invariant local 2nd-order area-metric actions

[JB, Dittrich & Krasnov 2023 — here Lorentzian signature]

1. **expansion:** $G = G(\eta) +$ fluctuations a
2. **"Ricci-Weyl" reparametrization:** $a_{\mu\nu\rho\sigma} \leftrightarrow (h_{\mu\nu}, \omega^{+}_{\mu\nu\rho\sigma}, \omega^{-}_{\mu\nu\rho\sigma}),\ \chi^{+}_{\mu\nu} = \overline{\chi^{-}_{\mu\nu}} \equiv \omega^{+}_{\mu\rho\nu\sigma}\frac{p^\rho p^\sigma}{p^2}$ transverse-traceless
3. **general ansatz:** $\mathscr{L}^{(2)}(a) = \mathscr{L}^{(2)}(h_{\mu\nu}, \chi^{+}_{\mu\nu}, \chi^{-}_{\mu\nu}) \supset 8$ tensorial structures
4. **linearized diffeomorphism invariance:** $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu\,,\ \ \chi^{\pm}_{\mu\nu} \to \chi^{\pm}_{\mu\nu}$

$$\mathscr{L}^{(2)}(a) = \mathscr{L}^{(2)}_{EH}(h) + \frac{1}{2}\sum_{\pm}\rho_{\pm}h_{\mu\nu}\chi^{\pm\mu\nu}p^2 + \frac{1}{2}(p^2 - m^2_{\pm})\chi^{\pm}_{\mu\nu}\chi^{\pm\mu\nu} \ \to\ \textbf{4 free parameters}\ (c_{+} = \overline{c_{-}}\ \ \forall c_{\pm} \in \mathscr{L})$$

Effective action for length-metric d.o.f.

$$\mathscr{L}^{(2)}_{eff}(h) = \mathscr{L}^{(2)}_{EH}(h) - \frac{1}{2}{}^{(1)}C_{\mu\nu\rho\sigma}(h)\left(\frac{\rho_+^2}{p^2 - m_+^2} + \frac{\rho_-^2}{p^2 - m_-^2}\right){}^{(1)}C^{\mu\nu\rho\sigma}(h) \quad \textbf{(linearized) quasi-local Einstein-Weyl action}$$

2-parameter subclass with shift-symmetric kinetic term ($\hat{=}$ continuum effective actions for spin foams)

$m_+^2 = m_-^2 \equiv m^2$

$\rho_+^2 + \rho_-^2 = 2$

$\rho_\pm^2 \leftrightarrow \gamma_\pm$

$i\mathscr{P}_{\mathscr{L}^{(2)}_{eff}(h)}{}^{\text{spin-2}} \propto \frac{i}{p^2} - \frac{i}{m^2}$ **ghostfree**

Hamiltonian analysis $\mathscr{L}^{(2)}(a)\Big|_{\text{shift-symmetric}}$ **:** **2 massless spin-2 modes (graviton)** + **5 massive modes**

γ**-dependent mixing of** $(+,\times)$ **polarizations for massless spin-2 mode** $\propto \boxed{\frac{\mathfrak{R}\left[\sqrt{\gamma_+}\right]\mathfrak{I}\left[\sqrt{\gamma_+}\right]}{m^2}}$

[JB, Dittrich & Krasnov 2023]

parity-violation effect attributed to γ
— towards GW signatures from area discreteness?

Summary

Take-home message

Area-metric gravity = EFT for the semiclassical regime of spin-foam QG (10 metric d.o.f. + 10 non-metric d.o.f.)

Future plan

Phenomenology associated with the presence of **non-metric d.o.f. & γ as parity-breaking parameter**

- **RG flow** for masses $m_{\pm}^2$ and γ [JB, Dittrich, Eichhorn & Schiffer w.i.p.]
- **black holes & mimickers**: static spherically symmetric solutions [JB & Dittrich w.i.p. + to do?!]
- implications of torsion-induced (?) parity breaking for cosmology

Viability as **quantum EFT**: unitarity,…

Classical stability at higher orders

Outlook 1: RG flow of Immirzi parameter and masses in area-metric gravity

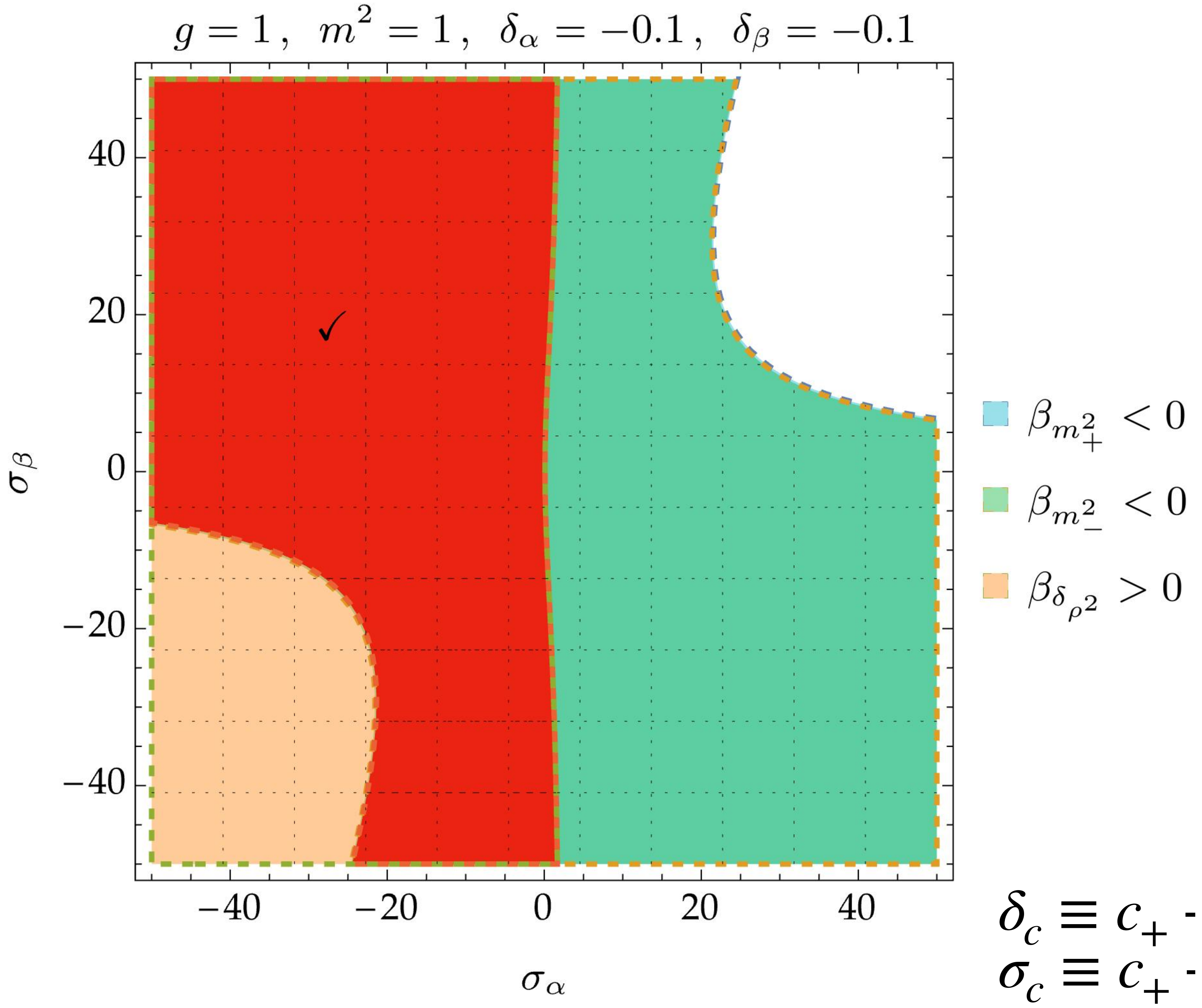
[JB, Dittrich, Eichhorn & Schiffer w.i.p.]

$$\Gamma_k = \Gamma_k^{(2)}[h_{\mu\nu}] + \int d^4x \sum_{\pm} \rho_{\pm k} \partial^\nu h^{\mu\rho} \partial^\rho \omega^{\pm}_{\mu\nu\rho\sigma} + \frac{1}{2}\partial_\alpha \omega^{\pm}_{\mu\nu\rho\sigma}\partial^\alpha \omega^{\pm\mu\nu\rho\sigma} + \frac{1}{2} m^2_{\pm k}\omega^{\pm}_{\mu\nu\rho\sigma}\omega^{\pm\mu\nu\rho\sigma} + \alpha_{\pm k} h \omega^{\pm}_{\mu\nu\rho\sigma}\omega^{\pm\mu\nu\rho\sigma} + \beta_{\pm k} h^{\mu\nu}h^{\rho\sigma}\omega^{\pm}_{\mu\rho\nu\sigma}$$

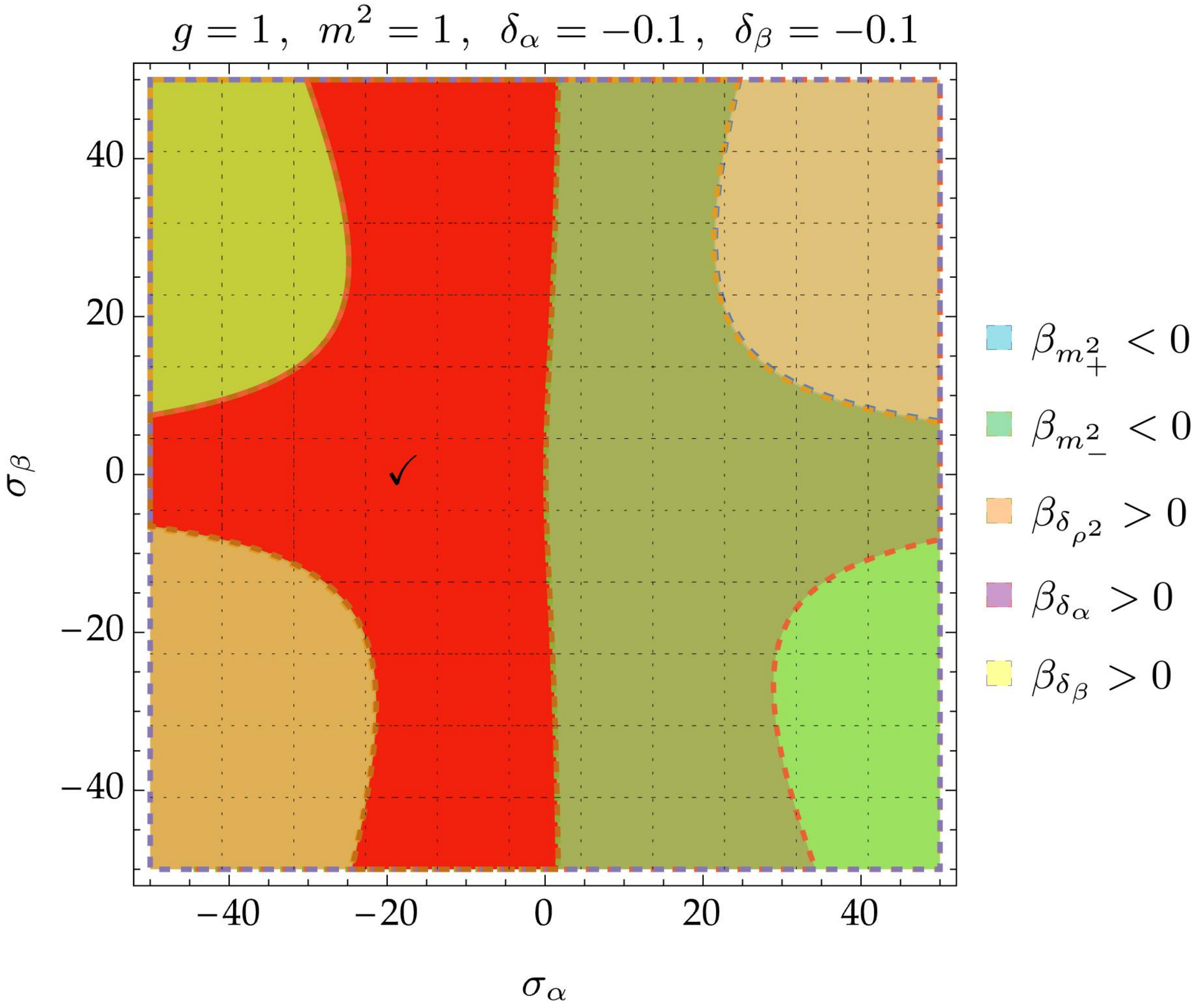
IR limit ($k \to 0$) wishlist — shift-symmetric subclass ($\rho_+^2 + \rho_-^2 = \text{const}$, $\delta_{\rho^2} \equiv \rho_+^2 - \rho_-^2 \propto \frac{1}{\gamma}$, $m_\pm^2 \equiv m^2$)

1. **GR: $m_\pm^2$ large ($\beta_{m_\pm^2} < 0$)**
2. **parity non-violation from γ:**
   $\delta_{\rho^2} \equiv \rho_+^2 - \rho_-^2 \to 0$ ($\beta_{\delta_{\rho^2}} > 0$)

β-functions expanded to $\mathcal{O}(m^0, \delta_{\rho^2}, \delta_\alpha^2, \delta_\beta^2, \delta_\alpha\delta_\beta)$:



$\delta_c \equiv c_+$ -
$\sigma_c \equiv c_+$ -



Outlook 2: Spherically symmetric solutions in higher-derivative quasi-local Einstein-Weyl gravity

[JB & Dittrich w.i.p.]

$$S[g] = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R - \mu C_{\mu\nu\rho\sigma}\frac{1}{\eta\Box + m^2}C^{\mu\nu\rho\sigma}\right) \rightarrow \textbf{localization}\text{ via } \psi_{\mu\nu\rho\sigma} \equiv -\left(\eta\Box + m^2\right)^{-1}C_{\mu\nu\rho\sigma}$$

static spherically symmetric ansatz: $ds^2 = -A(r)dt^2 + B(r)dt^2 + r^2 d\Omega^2$ + function $\psi(r)$

Weak-field limit

$A(r) = 1 + \delta a(r), B(r) = 1 + \delta b(r), \; \psi(r) = \delta c(r)$

2 free parameters for asymptotically flat solutions

$$A(r) = 1 - \frac{2M}{r} + C_{2-}\frac{e^{-\tilde{m}_2 r}}{r}, \quad \tilde{m}_2 = \sqrt{\frac{m^2}{2\mu - \eta}}$$

Yukawa corrections suppressed for $\mu \rightarrow \frac{1}{2}\eta$