CAUSAL EFFECTIVE FIELD THEORIES

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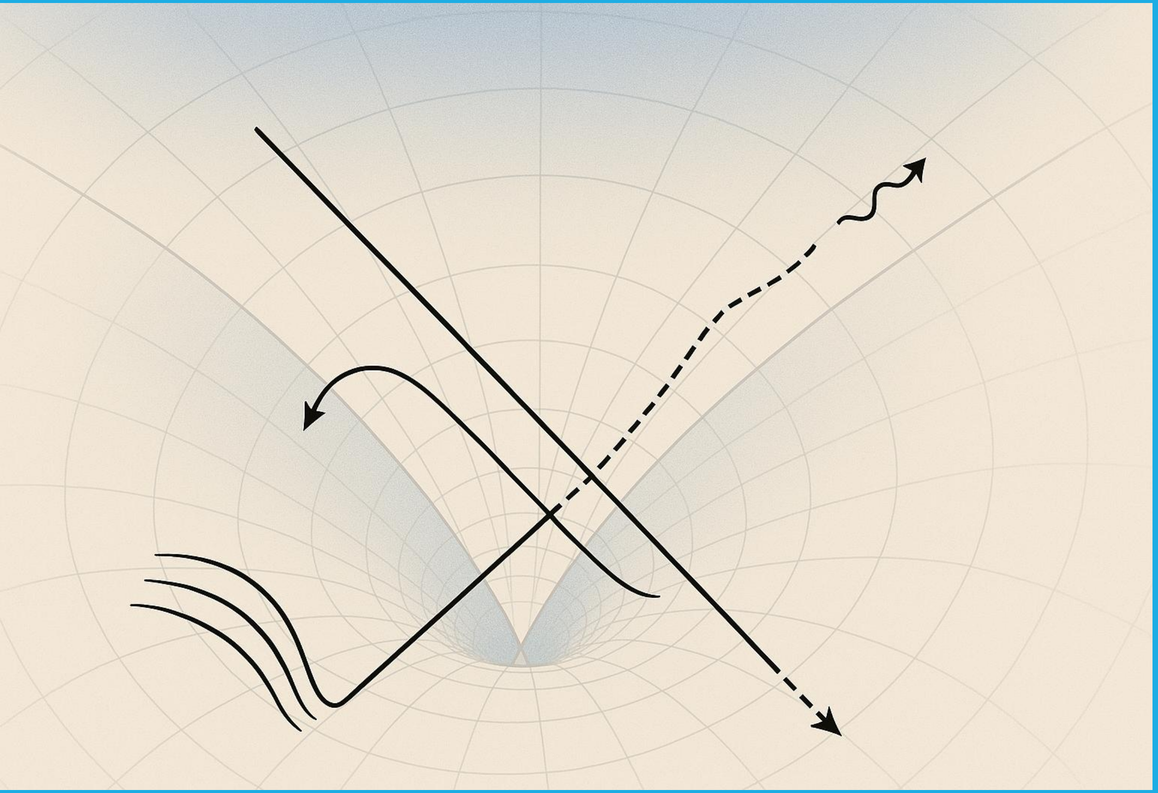
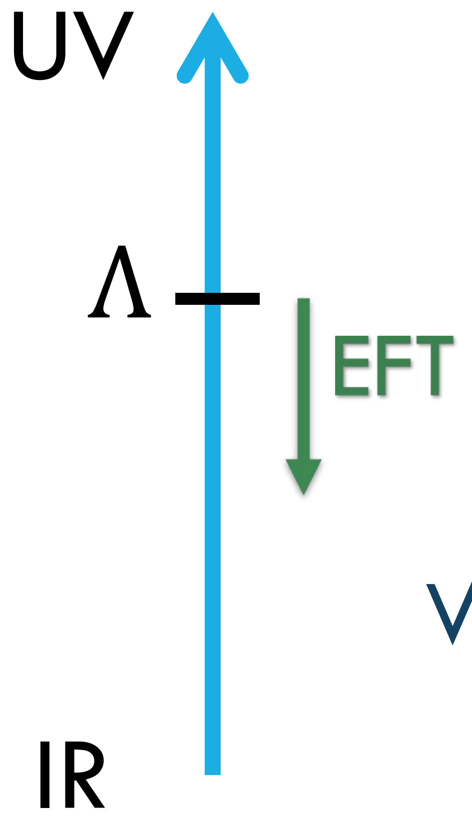
Quantum Spacetime and the Renormalization Group



IMPERIAL

EFFECTIVE FIELD THEORIES



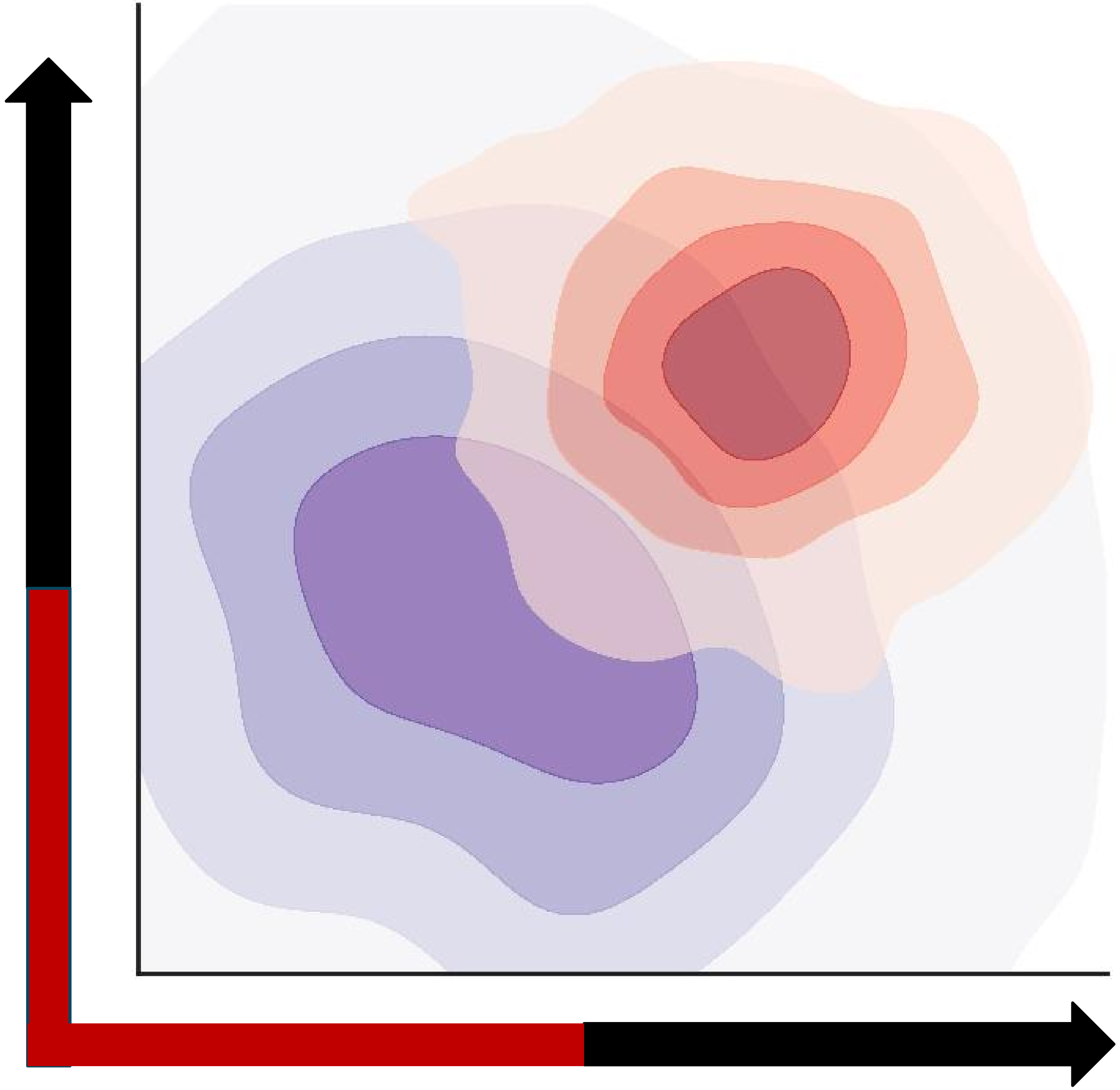
$$\mathcal{L} = \Lambda^4 \sum_n c_n \Lambda^{-n} \mathcal{O}_n$$

For example: $\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{c_8}{\Lambda^4}(\partial\phi)^4 + \ldots$

What are the values of the Wilson coefficients consistent with physical principles?

WHY DO THE VALUES OF WILSON COEFFICIENTS MATTER?

Theoretical priors can drastically
change the estimation of
parameters in a BSM model



UV = string theory, asymptotic safety, loop QG

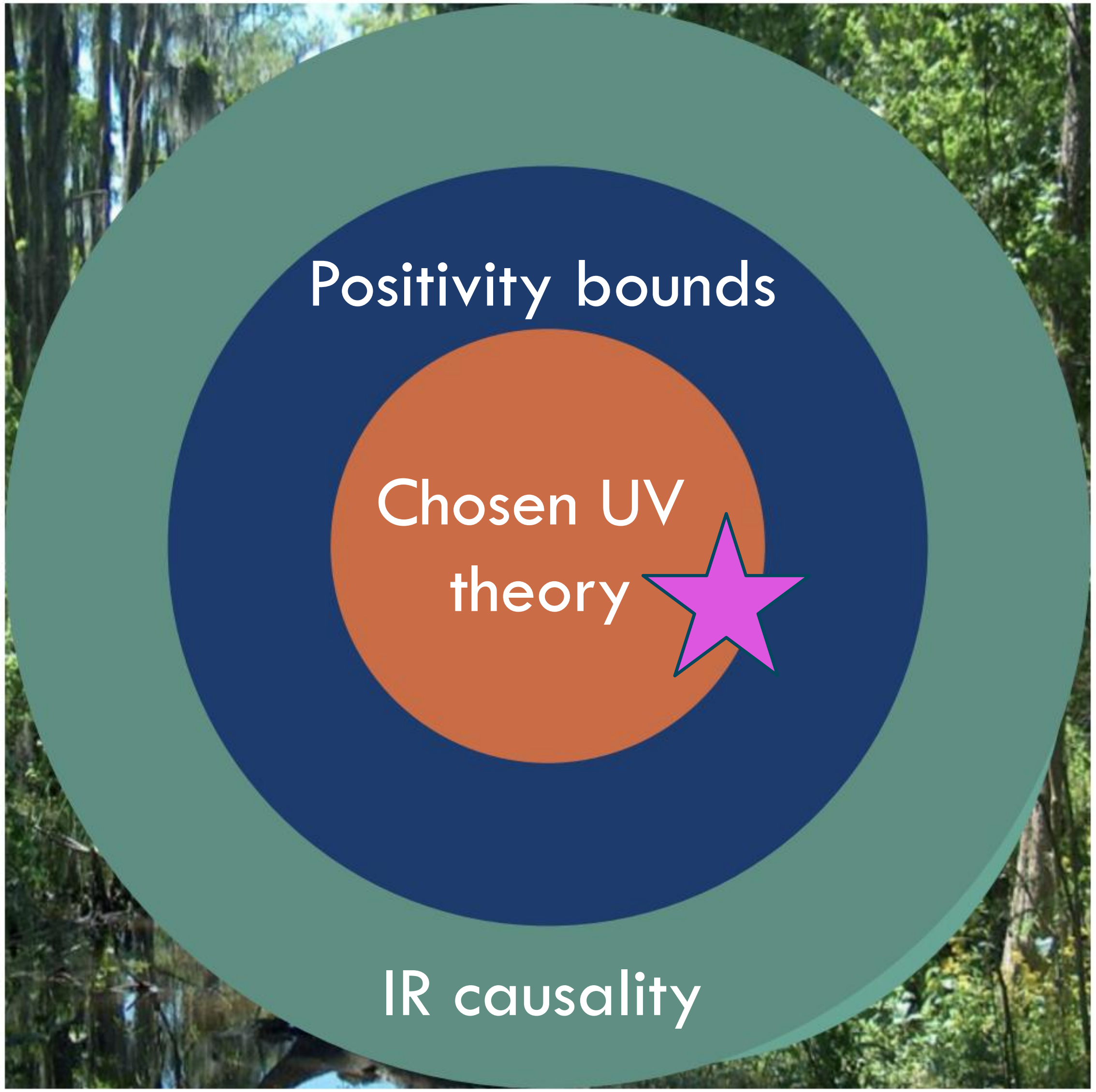
- **Swampland conjectures**

UV = local, unitary, causal, Lorentz invariant

- **Positivity bounds / S-matrix bootstrap** (Flat space, mostly 2-2 scattering)

Causal IR propagation

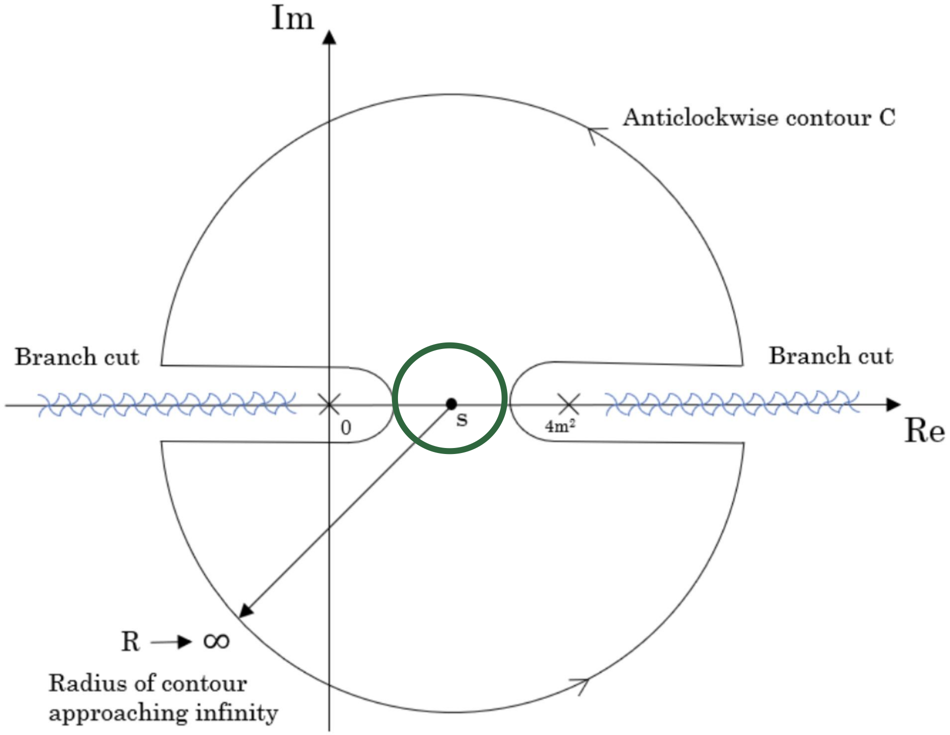
- **Causality bounds** (1-1 in any non-trivial background)



POSITIVITY BOUNDS

1 2 3 4

UV = local, unitary, causal, Lorentz invariant



1+3 ↓ , 2 ↓

$$A''(s)\Big|_{t=0} = \oint \frac{d\mu}{2\pi i}\frac{A(\mu)}{(\mu-s)^3} = \left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right)\frac{\operatorname{Im} A}{(\mu-s)^3} > 0$$

↑ related by 4

CAUSALITY BOUNDS ON FLAT SPACE

MICROCAUSALITY

$$[\mathcal{O}(x), \mathcal{O}(y)] = 0 \quad \text{for} \quad (x-y)^2 > 0$$

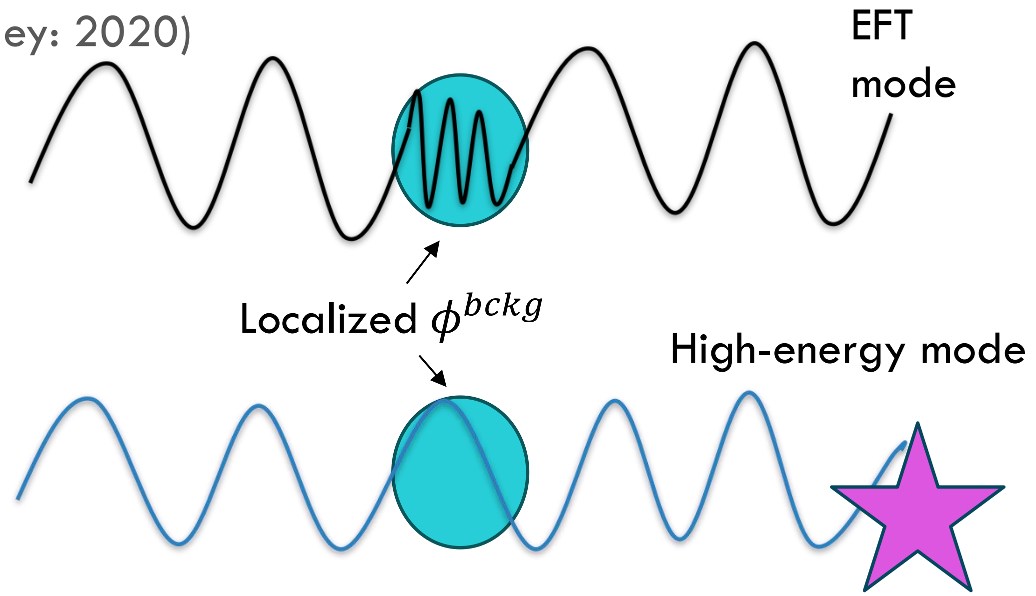
$$\Rightarrow \quad G_R(x,y) = 0 \quad \text{for} \quad (x-y)^2 > 0$$

Consider local propagation of information around fixed backgrounds and their implication on Wilson coefficients (Adams et al. 2006, de Rham, Tolley: 2020)

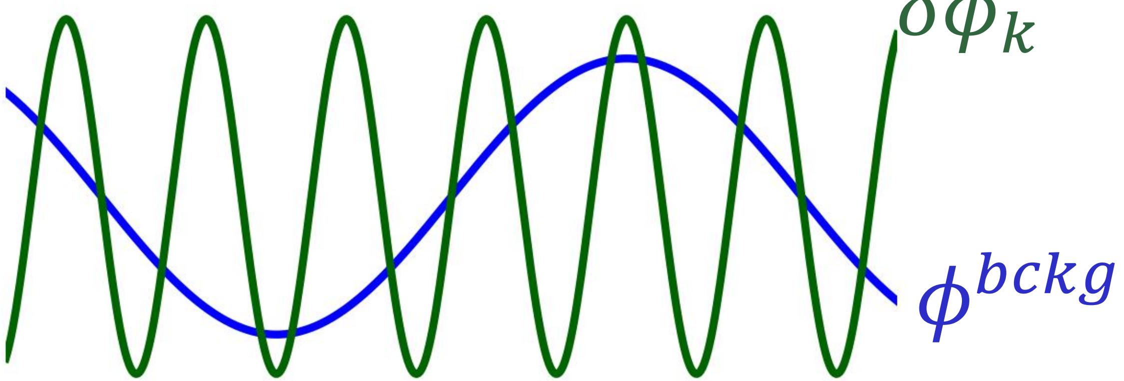
$$\phi = \phi^{bckg} + \delta\phi$$

In flat space, diagnose by looking at time delay:

$$\Delta T = -i \langle \text{in} | \hat{S}^\dagger \frac{\partial}{\partial \omega} \hat{S} | \text{in} \rangle$$



Work within the regime of validity of EFT and WKB approximation (for well-defined phase shift)



For a time-dependent background:

$$\delta\phi_k \sim e^{-ik\int c_s^{\text{eff.}}(t)\ dt}$$

$$G_{\text{ret}}(t,t') = \theta(t-t')\, i\left(\phi_k(t)\phi_k^*(t') - \phi_k(t')\phi_k^*(t)\right)$$

$$\Longrightarrow \quad |G_{\text{ret}}(t_1,t_2,k)| \leq e^{|\text{Im}\ k|\underbrace{\int_{t_1}^{t_2}(c_s^{\text{eff}}(k,\tilde{t})-1)\mathrm{d}\tilde{t}}_{\delta}} e^{|\text{Im}\ k|\ |t_2-t_1|}$$

$$\Delta T = 2\frac{\partial\delta}{\partial\omega}$$

Support of the Green's function

$$|G_{\text{ret}}(t_1, t_2, k)| \leq e^{|\text{Im}\ k| \int_{t_1}^{t_2} (c_s^{\text{eff}}(k,\tilde{t}) - 1) \text{d}\tilde{t}} e^{|\text{Im}\ k|\ |t_2 - t_1|}$$

Paley-Wiener theorem: Fourier Transform has compact support of radius $|t_1 - t_2|$ if:

$$|G_{\text{ret}}(t_1, t_2, k)| \leq C(D + |k|)^N e^{|\text{Im}\ k|\ |t_2 - t_1|}$$

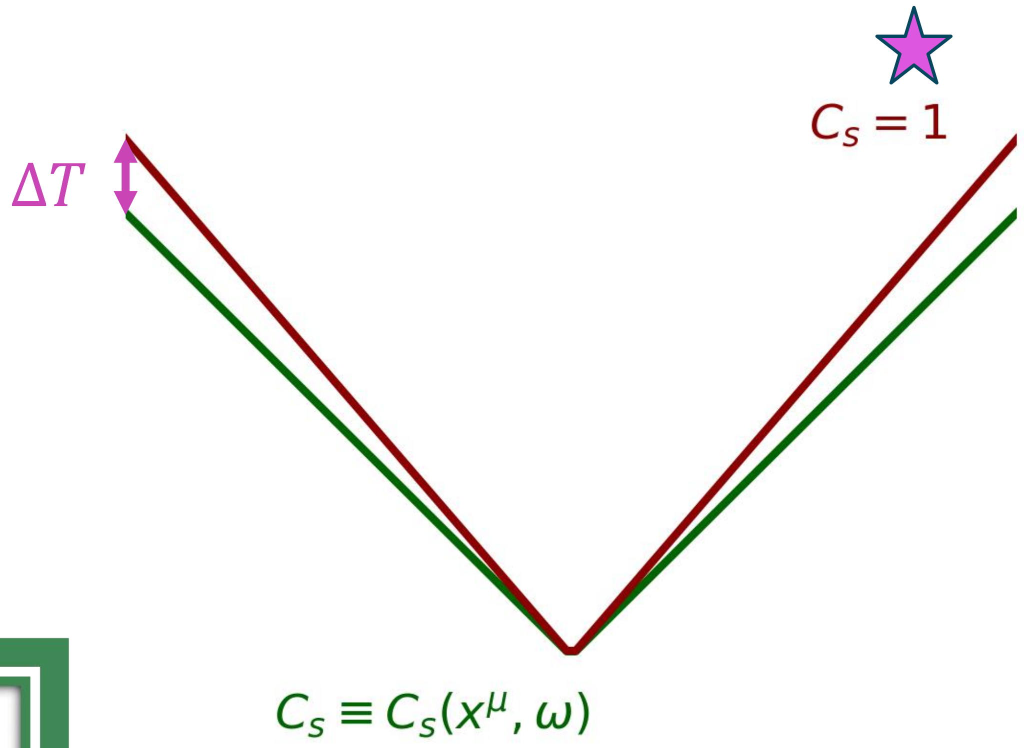
Implications on theories with broken Lorentz invariance (Hui, Nicolis, Podo, Zhou: 2025)

RESOLVABLE TIME DELAYS

$$\Delta T = 2\frac{\partial \delta}{\partial \omega}$$

Due to the uncertainty principle, time delays cannot be resolved if:

$$|\Delta T| \leq \lambda \sim \frac{1}{\omega}$$



CAUSALITY ⇒

$$\Delta T \gtrsim -1/\omega$$

(Eisenbud; 48, Wigner; 55)
(T. J. Hollowood and G. M. Shore: 2015)

RESOLVABLE TIME DELAYS

$$\Delta T = 2\frac{\partial \delta}{\partial \omega} \qquad \delta\phi \sim \delta\phi_0 \; e^{i\delta}$$

CAUSALITY

$$\Delta T \gtrsim -1/\omega$$

At leading order:

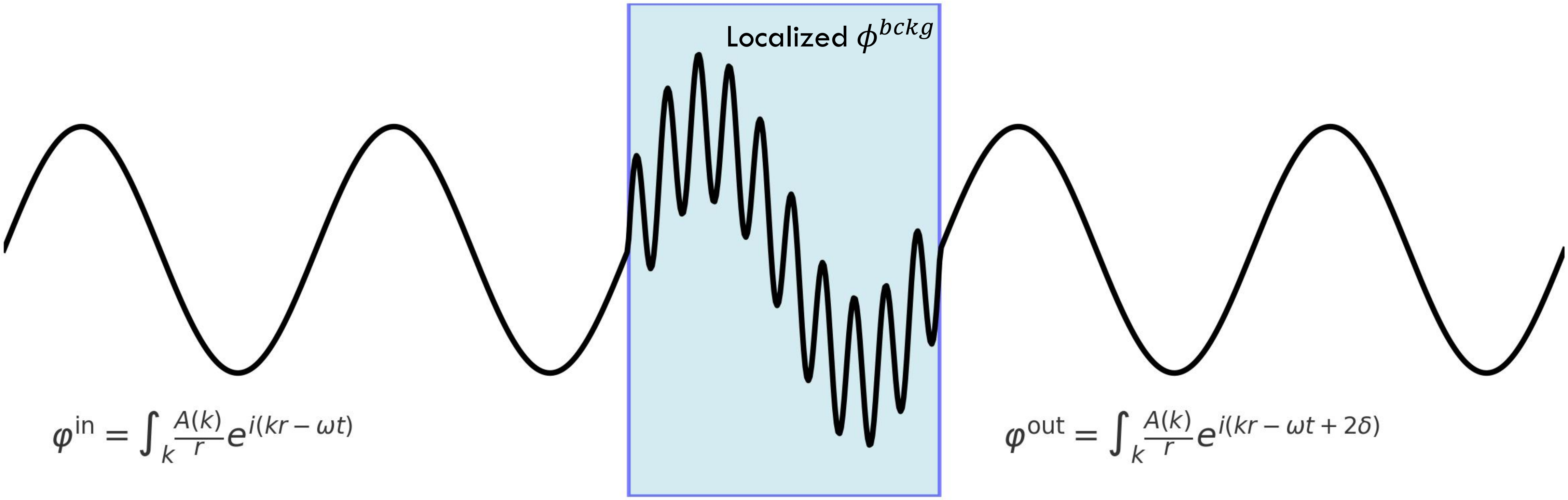
$$\frac{\lambda_{\text{background}}}{\lambda_{\text{perturbation}}} \int_{x \in \mathcal{M}} \left(1 - c_s(\lambda^{\text{pert.}})\right) \gtrsim -1$$

>>1 from WKB

<<1 from EFT

Acuasality needs $c_s > 1$ over large enough regions

TIME DELAYS AND SPATIAL SHIFTS



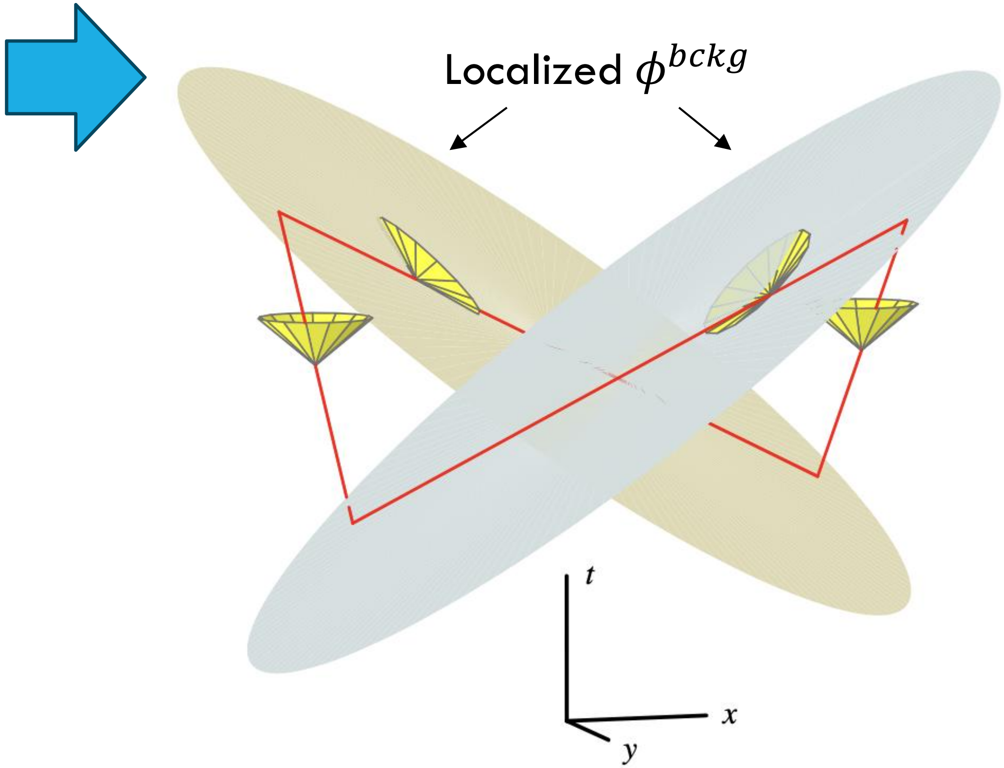
At fixed r, $\Delta T = 2\frac{\partial \delta}{\partial \omega}$ for ω conserved

At fixed t, $\Delta x = -2\frac{\partial \delta}{\partial k}$ for k conserved

If $\Delta T < -1/\omega$ or $\Delta x > 1/k$ the wave leaves the scatterer before it arrives to it

CLOSED TIMELIKE CURVES

$$\Delta T < -1/\omega$$



Chronology protection mechanism

(Kim, Thorne; 91, Hawking 92)

for EFTS:

Strong backreaction at the quantum level prevents the formation of CTCs

CTCs are not constructible in the regime of validity of EFT

(E. Babichev, V. Mukhanov, and A. Vikman; 2007, C. Burrage, C. de Rham, L. Heisenberg, and A. J. Tolley; 2011, D. E. Kaplan, S. Rajendran, F. Serra; 2024)

EXAMPLES

Scalar EFT:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_2}{2}(\partial_\mu\phi)^4$$

$$g_2 > 0$$

Same as simplest positivity bound:

$$\partial_s^2 \mathcal{A}(s) > 0$$

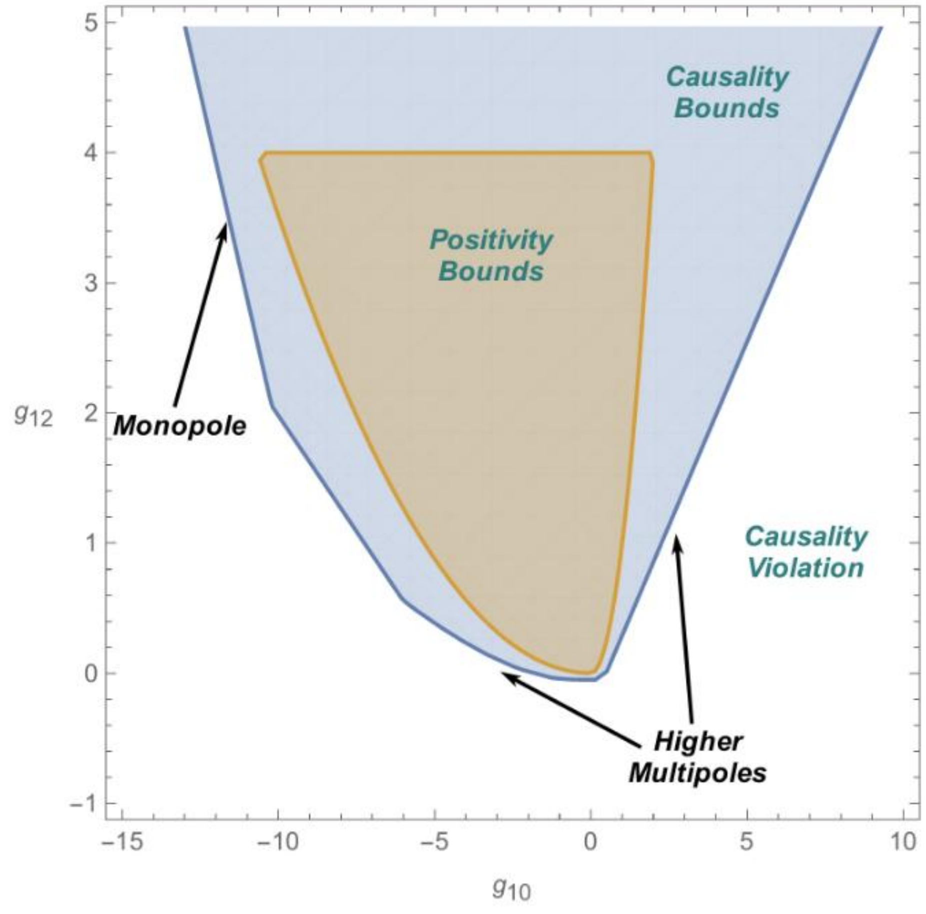
Photon EFT:

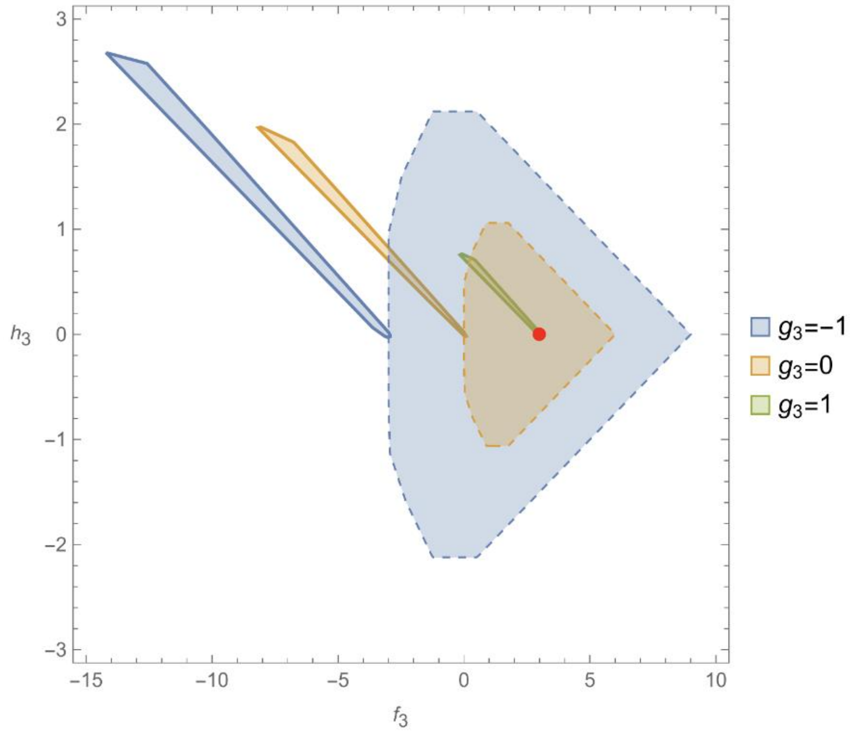
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

$$\alpha_1 \geq 0\,, \qquad \alpha_2 \geq 0$$

Adams et al. 2006 + verifications in many subsequent works

BOUNDS ON HIGHER ORDER OPERATORS





Photon EFT

$$\mathcal{A}_{++++} \supset \frac{f_3}{\Lambda^6} stu$$

$$\mathcal{A}_{+--+} \supset \frac{g_3}{\Lambda^6} s^3$$

$$\mathcal{A}_{+++-} \supset \frac{h_3}{\Lambda^6} stu$$

Scalar EFT

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{g_8}{\Lambda^4}(\partial\phi)^4 + \frac{g_{10}}{\Lambda^6}(\partial\phi)^2(\phi_{,\mu\nu})^2 + \frac{g_{12}}{\Lambda^8}\left((\phi_{,\mu\nu})^2\right)^2$$

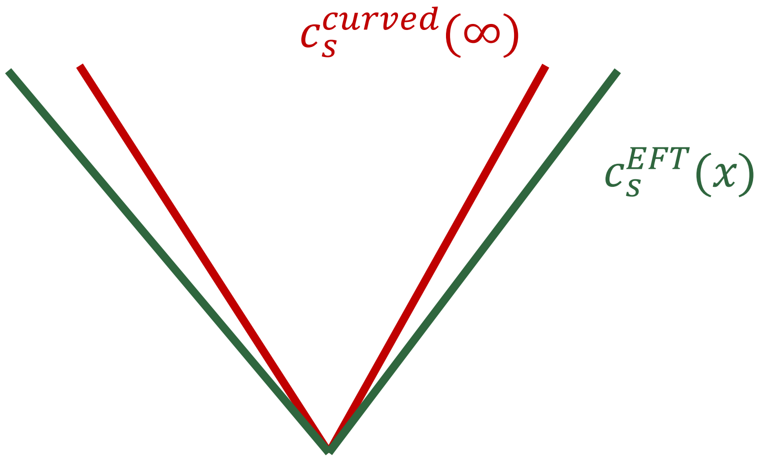
Causal propagation around any localized background that can be continuously deformed to the trivial one.

(MCG, C. de Rham , V. Pozsgay, A. Tolley; 2022) (MCG, C. de Rham, S. Jaitly, V. Pozsgay, A. Tokareva; 2023)

CAUSALITY BOUNDS ON CURVED SPACE

NOTIONS OF CAUSALITY ON CURVED BACKGROUNDS

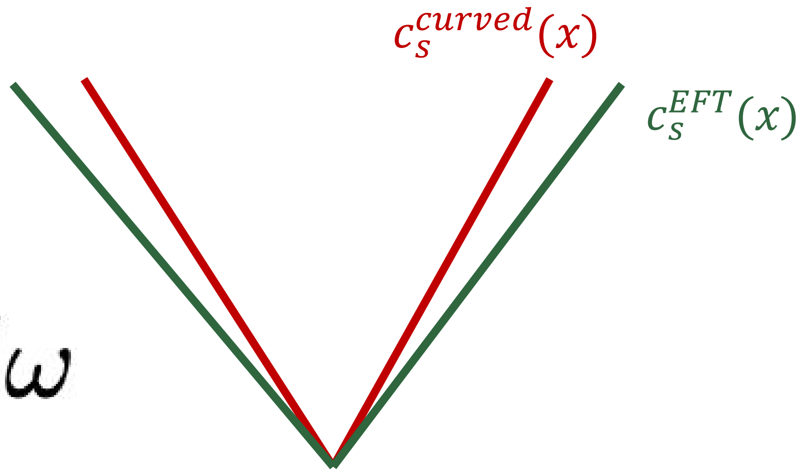
Asymptotic Causality (A. flat space)



$$\Delta T = \Delta T^{\mathrm{GR}} + \Delta T^{\mathrm{EFT}} \gtrsim -1/\omega$$

$$\Delta T^{\mathrm{GR}} = \lim_{\Lambda \to \infty} \Delta T$$

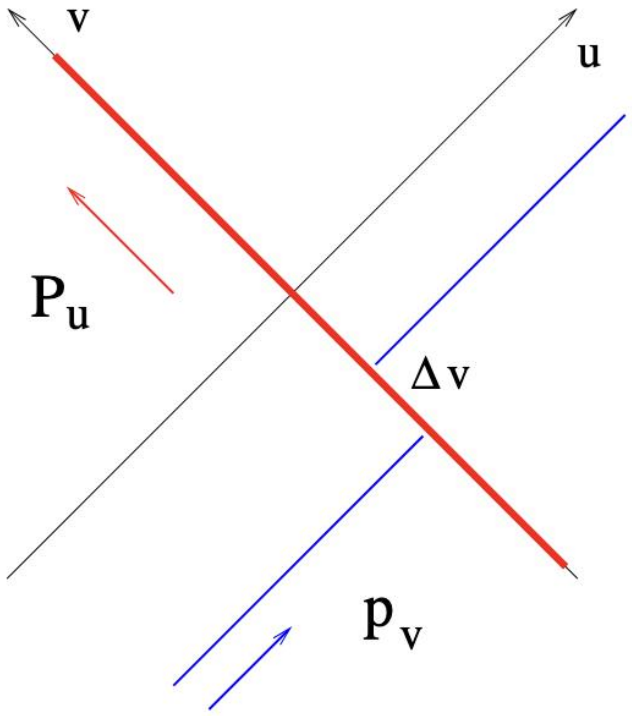
Infrared Causality



$$\Delta T^{\mathrm{EFT}} \gtrsim -1/\omega$$

$$c_n > -1/M_{Pl}$$

SHOCKWAVE BACKGROUNDS



$$ds^2 = -dudv + h(u, x_i)\, du^2 + \sum_{i=1}^{D-2} (dx_i)^2.$$

Related to eikonal limit of scattering amplitudes

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \alpha_3 W^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right)$$

T. Drummond and S. J. Hathrell; 80; X. O. Camanho, J. D. Edelstein, J. Maldacena, A. Zhiboedov; 2014, T. J. Hollowood and G. M. Shore; 2025, S. Cremonini, B. McPeak, Y. Tang; 2023, C. Y. R. Chen, C. de Rham, A. Margalit, and A. J. Tolley; 2025

PHOTON IN A CURVED BACKGROUND

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \alpha_3 W^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right)$$

Time delay for photon on charged shockwave background:

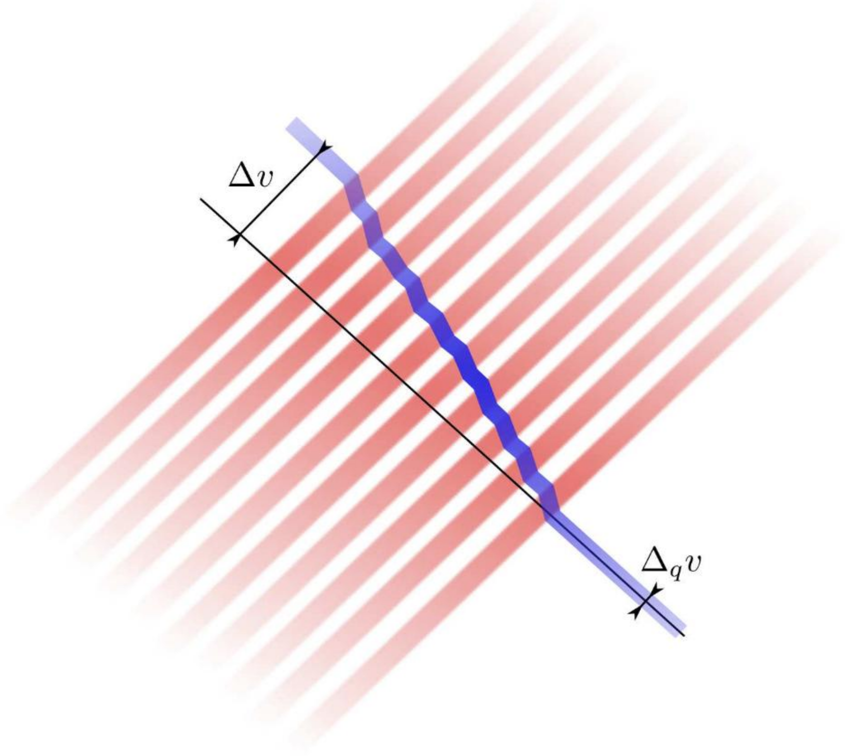
$$\Delta v = \underbrace{-\frac{3\pi q_0^2}{M_\mathrm{P}^2 \rho} - 16 \frac{m_0}{M_\mathrm{P}^2} \log \rho}_{\text{geometry}} + \underbrace{\alpha_i \frac{48\pi q_0^2}{\rho^3} \pm \alpha_3 \left(\frac{18\pi q_0^2}{M_\mathrm{P}^2 \rho^3} - \frac{64 m_0}{M_\mathrm{P}^2 \rho^2} \right)}_{\text{EFT}}$$

The log issue in 4D can be avoided by: 1) IR cutoff + A. Causality, 2) IR Causality, 3) negativity of derivative of time delay (T. Dray and G. 't Hooft; 85)

RESOLVABILITY

$$\Delta T_\ell^{\mathrm{EFT}} = \pm\alpha_3 \frac{2r_g}{b^2\Lambda^2} \ll \sqrt{\frac{r_g}{b}}\omega^{-1} \ll \omega^{-1}$$

$\alpha_3 \to \alpha_3/\Lambda^2$ Validity of EFT : $\omega \ll \Lambda^2\sqrt{\frac{b}{r_g}}$



Time delay is unresolvable within the regime of validity of the EFT even when multiple shockwaves are stacked (C. Y.-R. Chen, C. de Rham, A. Margalit, A. J. Tolley; 2024)

CAUSAL PHOTONS IN A CURVED BACKGROUND

Weak Gravity Conjecture

Weakened positivity ($c_n < -1/M_{Pl}$): small violations of WGC are consistent with unitarity and causality (L. Alberte, C. de Rham, S. Jaitly and A.J. Tolley; 2020, Henriksson, B. McPeak, F. Russo and A. Vichi; 2022)

Quasi-normal modes

Causal EFT corrections make quasi-normal mode perturbations decay faster. (Melville; 2024)

(A)DS BACKGROUNDS

AdS

- Causality implies that the geodesics between boundary points lie at the boundary (Gao, Wald; 2000)
- Bulk Causality = ANEC of the boundary (W.R. Kelly and A.C. Wall; 2014)
- Commutativity of shockwave implies bounds on the EFTs (M. Kologlu, P. Kravchuk, D. Simmons-Duffin and A. Zhiboedov; 2019)

dS

- The "fastest geodesics" (reach the future boundary with the largest positive spatial shift) is the one with the largest impact parameter (N. Bittermann, D. McLoughlin and R.A. Rosen; 2022)
- IR causality: The reference lightcone is the one of a minimally coupled high-energy particle in dS (de Rham, Tolley; 2020, Carrillo-Gonzalez; 2023)

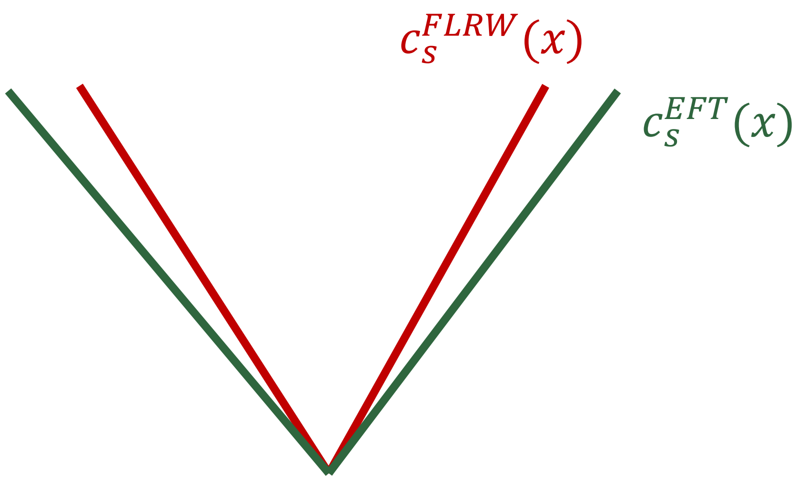
CAUSALITY IN FLRW

$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

$$\frac{k}{a(t_f)}(a(t_f)\Delta r) \sim k \int_{t_i}^{t_f} \frac{dt}{a(t)} \left(c_s^{\mathrm{EFT}}(k,t) - c_s^{\mathrm{FRW}}(k,t)\right) < 1$$

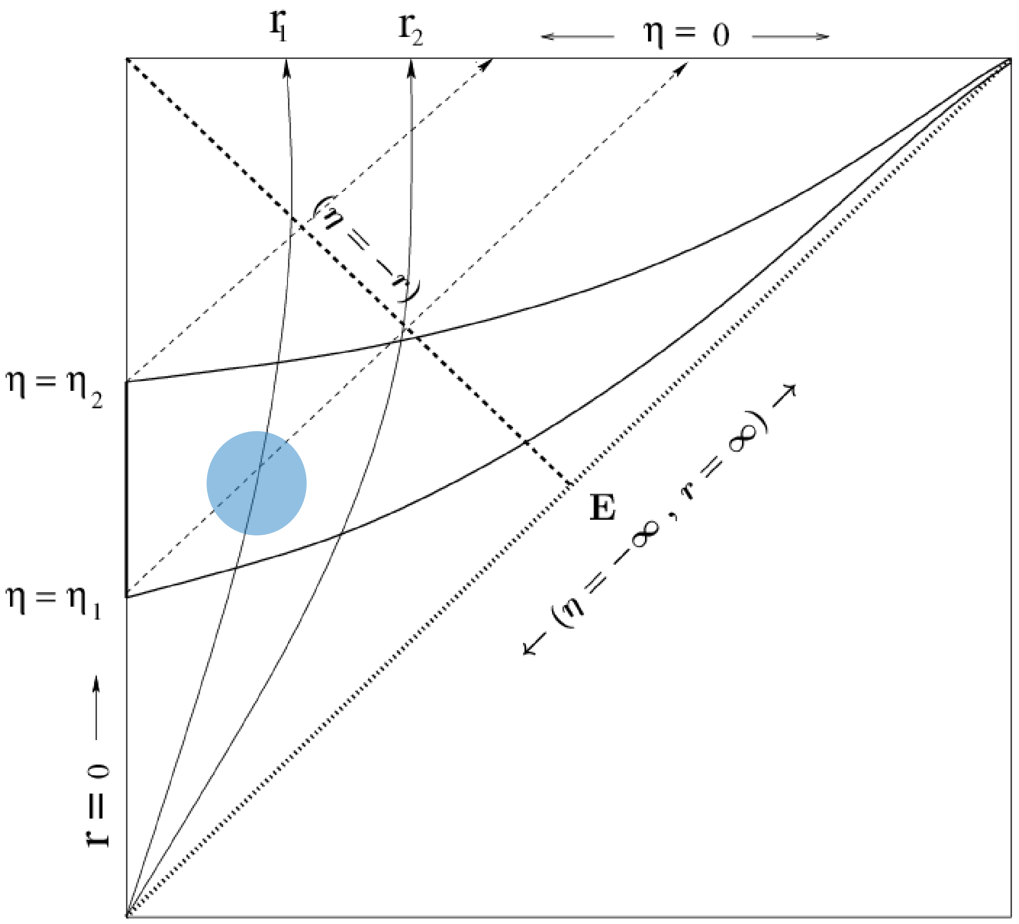
Causal EFTs have negative physical spatial shifts, up to a positive unresolvable contribution.

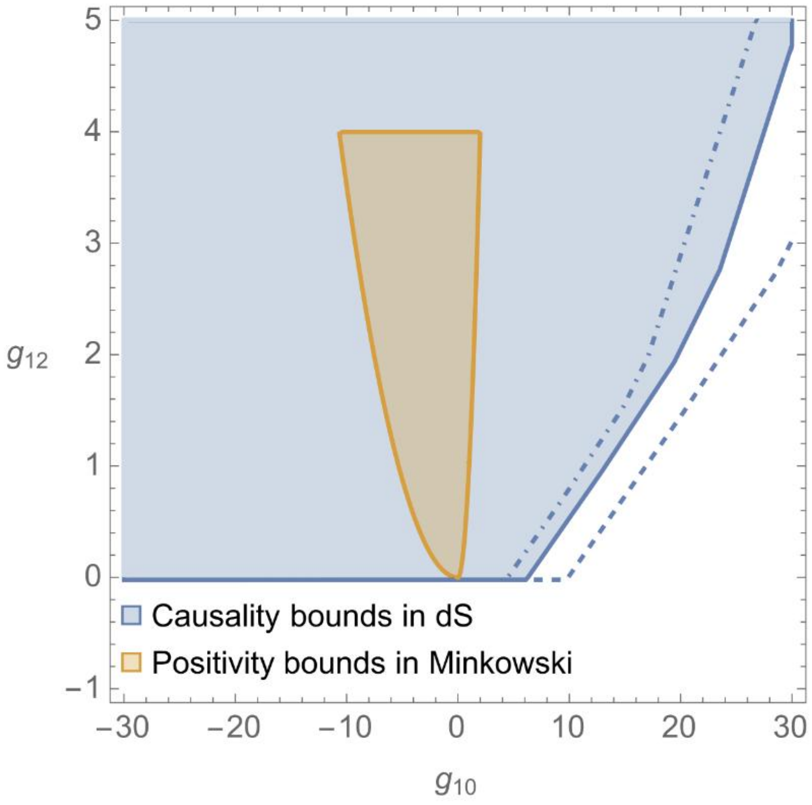
→ EFT modes should not travel outside of the particle horizon.



Scalar EFT in de Sitter

$$\mathcal{L} = -\frac{1}{2}(\nabla\phi)^2 + \frac{g_8}{\Lambda^4}(\nabla\phi)^4 + \frac{g_{10}}{\Lambda^6}(\nabla\phi)^2(\phi_{;\mu\nu})^2 + \frac{g_{12}}{\Lambda^8}\left((\phi_{;\mu\nu})^2\right)^2$$





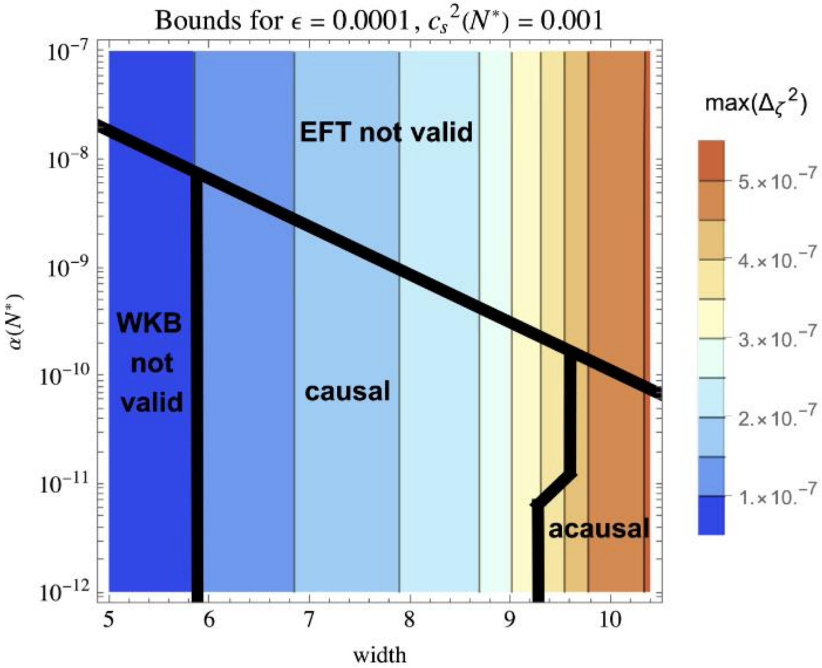
$$\mathcal{L}^{\rm HO} = \frac{h_{12}}{M^8}\left((\phi_{;\mu})^2\right)^3 \qquad h_{12} < 1.64$$

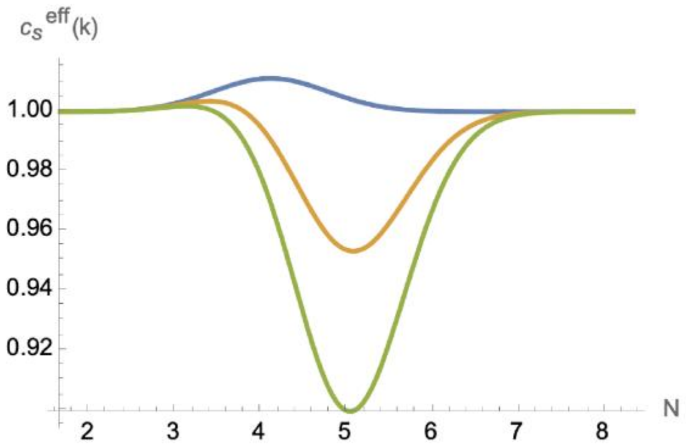
(MCG; 2023)

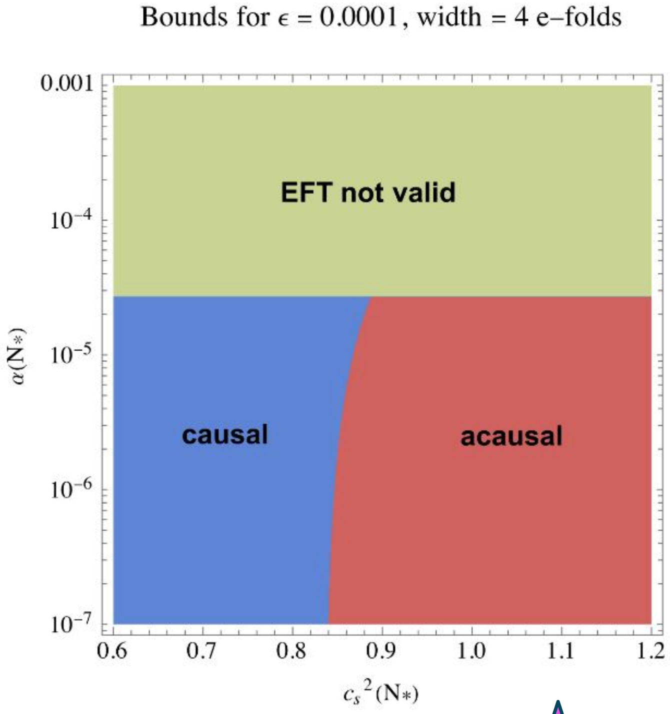
EFT OF INFLATION

$$(c_s^{\text{eff}}(k,N))^2 = c_s^2(N) + \alpha(N)\frac{k^2}{a^2H^2}$$

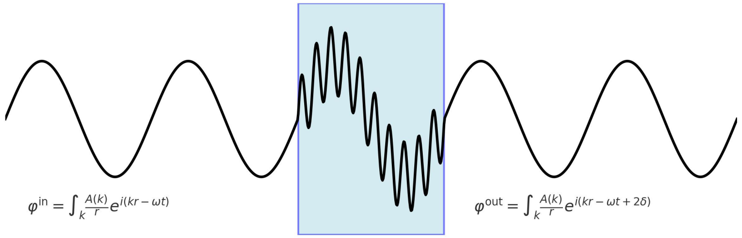
Bounds on time dependence of Wilson coefficients

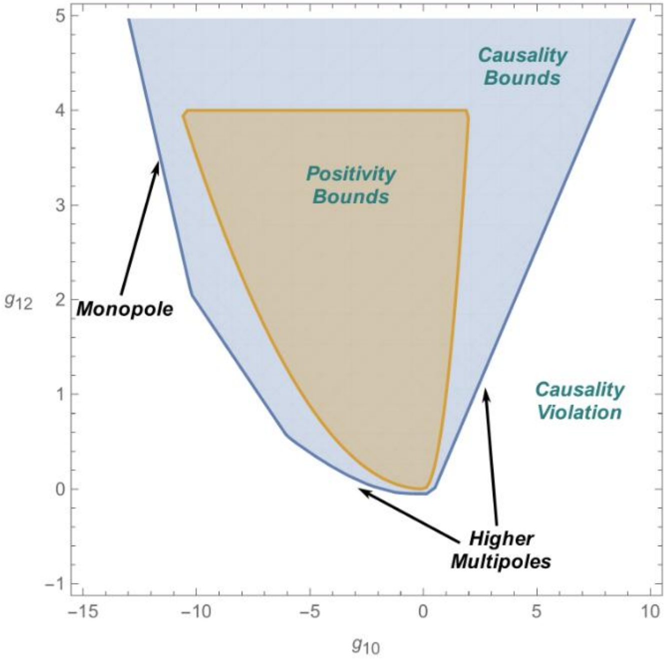


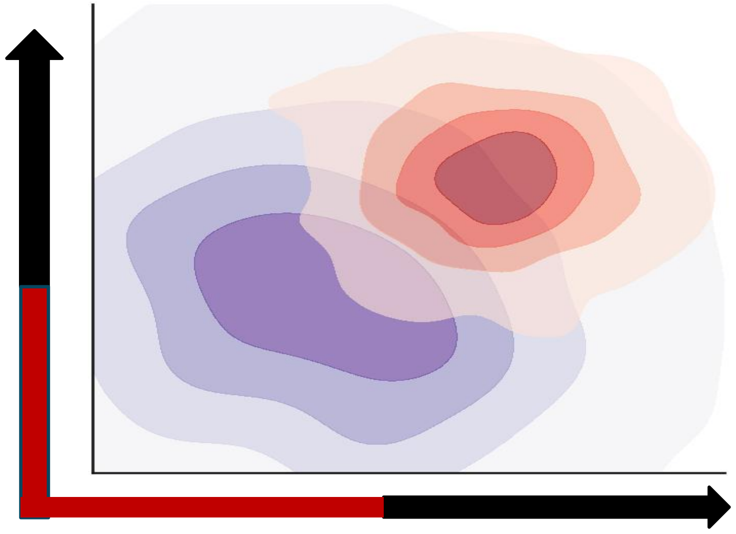




(MCG, S. Cespedes; 2025)







CAUSALITY BOUNDS ON EFTS

BACK-UP SLIDES

ASSUMPTIONS

Property	Causality Bounds	Positivity Bounds
Lorentz invariance	• Lorentz invariant EFT	• Invariant EFT and UV completion • Crossing symmetry
Unitarity	• Hermitian Hamiltonian: real Wilson coefficients	• Positive discontinuity of the EFT and UV amplitude
Causality	• No resolvable time advance	• Analyticity of amplitude in the complex s plane for fixed t
Locality	• IR theory is local	• IR and UV theories are local • Froissart-like bound in the UV
Other assumptions	• EFT and WKB expansions under control • Background generated by localized external source	• IR EFT is under perturbative control