An asymptotically free bosonic field theory in 4 dimensions

Răzvan Gurău

(Quantum spacetime and the Renormalization Group)

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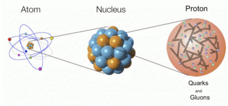
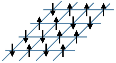
Fundamental interactions in nature (electroweak and strong interaction)



Condensed matter (Ising spins, Fermi liquids, etc.)



And quantum gravity (AdS/CFT, asymptotic safety).

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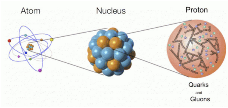

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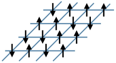

And quantum gravity (AdS/CFT, asymptotic safety).

Best prediction in physics: electron anomalous magnetic moment
0.001 159 652 18(1)

But must be handled with care: naively overestimates the cosmological constant 10^{120} times

Weak coupling

Perturbation theory

$\rightarrow$ electron anomalous magnetic moment

Strong coupling

?

- functional RG (derivative expansion): no small parameter
- $4 - \epsilon$ expansion: ϵ not small
- lattice (numerics): refinement limit

Understand strongly coupled quantum field theories

Conformal Field Theories (CFT)

Solvable strongly coupled theories $\sim$ conformally invariant (1980s onward)

- fixed points of the renormalization group, phase transitions
- universal (same CFT for many physical systems)

Limited $\rightarrow$ perturbations **break conformal invariance**

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Do more!

Strongly coupled theories **outside** the conformal limit

Vector with N components (spherical model, $O(N)$ model)

Matrix with N^2 entries (Quantum Chromodynamics $SU(N)$, $N = 3$)

Access the strong coupling regime in a $1/N$ expansion!

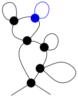
Vectors – large N limit solvable but too restrictive in any dimension [Berlin, Kac '52; Stanley '68;...]

Matrices – planar limit very difficult in more than zero dimensions ['t Hooft '74;...]

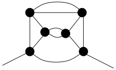
Tensors – melonic limit in between non trivial but accessible in any dimension

[Sasakura '90; Ambjørn et al. '90; Boulatov '92; Ooguri '92,... Gurau '10 '11; Rivasseau '11, Bonzom '11,... Witten '16; Klebanov et al. '17,'18; Minwalla et al. '17; Tseytlin et al. '17; Ferrari et al. '17, ... Benedetti Harribey '19 ...]

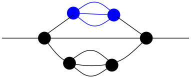
Vectors – simple
"snails": local insertions

Matrices – complicated
planar



Tensors – in between
"melonic": recursive bilocal insertions

A Tensor Field Theory:

- solvable at leading large N order
- asymptotically free (strongly coupled in the infrared)
- stable

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Coupled to gravity – see poster by Zois Gyftopoulos

An apology for the free theory – the Gaußian free field

$$\langle \phi(x)\phi(y)\rangle = C_{xy} = \frac{1}{|x-y|^{2\Delta_\phi}}\,, \qquad S_0 = \frac{1}{2}\int d^d x\,\phi(x)(-\partial_\mu\partial^\mu)^{\frac{d}{2}-\Delta_\phi}\phi(x)$$

It is not enough to know the correlations of ϕ, we need the correlations of arbitrary local operators

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Renormalized operators by Wick ordering

$$:\phi^n(x) := e^{-\frac{1}{2}\int_{x,y}\frac{\delta}{\delta\phi_x} C_{xy}\frac{\delta}{\delta\phi_y}}\phi^n(x) = \phi^n(x) - \binom{n}{2}\phi^{n-2}(x)C_{xx} + \dots \ , \qquad \langle :\phi^n(x):\rangle = 0 \ ,$$

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CFT at fixed point, but you need OPE coefficients:

$$\langle :\phi^{n_1}(x)::\phi^{n_2}(y)::\phi^{n_3}(z):\rangle = \left[e^{\frac{\delta}{\delta\phi^1}C\frac{\delta}{\delta\phi^2}+\frac{\delta}{\delta\phi^1}C\frac{\delta}{\delta\phi^3}+\frac{\delta}{\delta\phi^2}C\frac{\delta}{\delta\phi^3}}(\phi^1_x)^{n_1}(\phi^2_y)^{n_2}(\phi^3_z)^{n_3} \right]_{\phi=0}$$

$$= \frac{n_1!n_2!n_3!\left[\left(\frac{n_1+n_2-n_3}{2}\right)!\left(\frac{n_1+n_3-n_2}{2}\right)!\left(\frac{n_2+n_3-n_1}{2}\right)!\right]^{-1}}{|x-y|^{(n_1+n_2-n_3)\Delta_\phi}|x-z|^{(n_1+n_3-n_2)\Delta_\phi}|y-z|^{(n_2+n_3-n_1)\Delta_\phi}}$$

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$$= \frac{n_1!n_2!n_3!\left[\left(\frac{n_1+n_2-n_3}{2}\right)!\left(\frac{n_1+n_3-n_2}{2}\right)!\left(\frac{n_2+n_3-n_1}{2}\right)!\right]^{-1}}{|x-y|^{(n_1+n_2-n_3)\Delta_\phi}|x-z|^{(n_1+n_3-n_2)\Delta_\phi}|y-z|^{(n_2+n_3-n_1)\Delta_\phi}}$$

There is work to do even in free theories...

$$\mathcal{Z} = \int \mathcal{D}\psi \; e^{-S(\psi)} \; , \qquad S(\psi) = S_0(\psi) + V(\psi)$$

$$S_0(\psi) = \frac{1}{2}\int d^d x \; \psi(x)(-\partial_\mu \partial^\mu)\psi(x) \; , \qquad V(\psi) = \sum_i \lambda_i \int d^d x \; \partial^{p_i} \psi^{n_i}(x)$$

Wilson's renormalization group

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The canon

Decompose $\psi = \phi + \chi$ with:

- low modes $\phi(x) = \int_{|p|\leq\Lambda} e^{\imath p\cdot x}\psi(p)$
- fluctuations $\chi(x) = \int_{|p|>\Lambda} e^{\imath p\cdot x}\psi(p)$

Integrate the fluctuations to get effective action:

$$\mathcal{Z} = \int_{|p|\leq\Lambda} \mathcal{D}\phi\, e^{-S^\Lambda(\phi)}\,, \qquad \underbrace{S^\Lambda(\phi)}_{\text{Effective action at scale }\Lambda} = S_0(\phi) - \ln \int_{|p|>\Lambda} \mathcal{D}\chi\, e^{-S_0(\chi)+V(\phi+\chi)}$$

Low energy correlations:

$$\langle\phi(x_1)\ldots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\psi\, e^{-S(\psi)}\phi(x_1)\ldots\phi(x_n) = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\, e^{-S^\Lambda(\phi)}\phi(x_1)\ldots\phi(x_n)\,.$$

Beta functions

$$S^\Lambda(\phi) = \frac{Z_\Lambda}{2}\int d^dx\,\phi(-\partial_\mu\partial^\mu)\phi + \sum_i \Lambda^{d-\overbrace{(n_i\Delta_\phi + p_i)}^{\Delta_i}} g_i(\Lambda)\; Z_\Lambda^{\frac{n}{2}} \int d^dx\,\partial^{p_i}\phi^{n_i}(x)$$

with $\Delta_\phi = \frac{d-2}{2}$ the canonical field dimension:

$$\phi(x) = \Lambda^{\Delta_\phi}\tilde{\phi}(\tilde{x})\big|_{\tilde{x}=\Lambda x}\,, \qquad \partial^{p_i}\phi^{n_i}(x) = \Lambda^{\Delta_i}\partial_{\tilde{x}}^{p_i}\tilde{\phi}^{n_i}(\tilde{x})\big|_{\tilde{x}=\Lambda x}$$

Evolution with Λ captured by beta functions and anomalous field dimension:

$$\gamma_\phi = -\frac{1}{2}\Lambda\partial_\Lambda \ln Z_\Lambda\,, \qquad \beta_i = \Lambda\partial_\Lambda g_i = -(d - \Delta_i - n_i\gamma_\phi)g_i + O(g^2)$$

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... and Callan-Symanzik equations for the correlations of the renormalizaed field $\varphi = \sqrt{Z_\Lambda}\phi$:

$$G(x_1,\dots x_n) = \langle\varphi(x_1)\dots\varphi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int\mathcal{D}\phi\; e^{-S^\Lambda(\phi)}\sqrt{Z_\Lambda}\phi(x_1)\dots\sqrt{Z_\Lambda}\phi(x_n)\,,$$

$$\left(\Lambda\frac{\partial}{\partial\Lambda} + \beta_i\frac{\partial}{\partial g_i} + n\gamma_\phi\right)G = 0\,, \qquad G(x_1,\dots x_n) = \Lambda^{n\Delta_\phi}\tilde{G}(\tilde{x}_i, g)\Big|_{\tilde{x}_i=\Lambda x_i}$$

Fixed points

Fixed point $g_i^\star$ such that $\beta_j(g^\star) = 0$, hence $\gamma_\phi(g^\star) = \gamma_\phi^*$

$$g_i = g_i^\star + h_i \;, \quad \Lambda \partial_\Lambda h_i = (\partial_{g_j} \beta_i)_\star \, h_i \Rightarrow h = \underbrace{(S^{-1} \Lambda^{-\theta} S)}_{\Lambda^{(\partial_{g_j}\beta_i)_\star}} h \Rightarrow \begin{cases} \mathrm{Re}(\theta) > 0 \;, & \text{relevant} \\ \mathrm{Re}(\theta) < 0 \;, & \text{irrelevant} \\ \mathrm{Re}(\theta) = 0, \mathrm{Im}(\theta) \neq 0 \;, & \text{limit cycle} \\ \theta = 0 \;, & \text{marginal} \end{cases}$$

Stability analysis of marginal couplings is more involved!

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$$G_\star(x_1, \dots x_n) = \Lambda^{n\Delta_\phi} \tilde{G}(\tilde{x}_i, g^\star)\Big|_{\tilde{x}_i = \Lambda x_i} \;, \qquad \left(\Lambda \frac{\partial}{\partial\Lambda} + n\gamma_\phi^\star\right) G_\star = 0$$

for instance:

$$\left(\Lambda \frac{\partial}{\partial\Lambda} + 2\gamma_\phi^\star\right) \Lambda^{2\Delta_\phi} \tilde{G}^{(2)}(\Lambda(x-y), g^\star) = 0 \Rightarrow G_\star^{(2)}(x,y) = \frac{f(g^\star)}{|x-y|^{2\Delta_\phi + 2\gamma_\phi^*}}$$

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And then there are the other scaling (primary) operators, OPE expansions, etc...

The ϕ^4 model

$$S(\psi) = \int d^d x \left(\frac{1}{2}\partial_\mu \psi \partial^\mu \psi + \frac{-CT(\lambda) + \kappa}{2}\psi^2 + \frac{\lambda}{4!}\psi^4 \right)$$

Counterterm tunes to criticality $\langle \psi\psi \rangle\big|_{\kappa=0, p=0} = 0$ and κ is perturbation with respect to the critical theory

Compute in perturbations: $S_\Lambda^{(2)} = \kappa - \kappa\lambda + (-\Delta)\left(1 - \lambda^2 \partial_{-\Delta}\right) + O(\Delta^2)\,, \qquad S_\Lambda^{(4)} = \lambda - \lambda^2$

Project on local operators: $S^\Lambda(\phi) = \int d^d x \left(\frac{Z_\Lambda}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2!}\mu^{d-2\Delta_\phi} g_2(\Lambda) Z_\Lambda \phi^2 + \frac{1}{4!}\mu^{d-4\Delta_\phi} g_4(\Lambda) Z_\Lambda^2 \phi^4 \right) + CT + \text{irrelevant}$

$$Z_\Lambda = 1 - \lambda^2 \partial_{-\Delta}\,, \quad g_2 = \mu^{-(d-2\Delta_\phi)} \frac{\kappa - \kappa\lambda}{Z_\Lambda}\,, \quad g_4 = \mu^{-(d-4\Delta_\phi)} \frac{\lambda - \lambda^2}{Z_\Lambda^2}$$

$$\gamma_\phi = A\, g_4^2\,, \qquad \beta_2 = -(d - 2\Delta_\phi - 2\gamma_\phi)\, g_2 + B\, g_2\, g_4\,, \qquad \beta_4 = -(d - 4\Delta_\phi - 4\gamma_\phi)\, g_4 + C\, g_4^2$$

$A, B, C > 0$

Fixed points in the ϕ^4 model

$$O(g^2): \qquad \gamma_\phi = A\, g_4^2\,, \qquad \beta_2 = -(d-2\Delta_\phi)\, g_2 + B\, g_2\, g_4\,, \qquad \beta_4 = -(d-4\Delta_\phi)\, g_4 + C\, g_4^2$$

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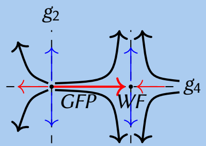
$d = 4-\epsilon\ (\Delta_\phi = 1-\frac{\epsilon}{2}) \qquad \gamma_\phi = A\, g_4^2\,,\ \ \beta_2 = -2g_2 + B\, g_2\, g_4\,,\ \ \beta_4 = -\epsilon g_4 + C\, g_4^2$

- Gaußian $(g_4^\star, g_2^\star\ |\gamma_\phi^\star) = (0, 0\ |0)$, two relevant directions $\theta_{\phi^4} = \epsilon$, $\theta_{\phi^2} = 2$
- Wilson Fisher $(g_4^\star, g_2^\star\ |\gamma_\phi^\star) = (\frac{1}{C}\epsilon, 0\ |\frac{A}{C^2}\epsilon^2)$, one relevant $\theta_{\phi^2} = 2 - \frac{B}{C}\epsilon$ and one irrelevant $\theta_{\phi^4} = -\epsilon$,



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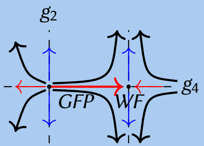

$d = 4$ ($\Delta_\phi = 1$) $\quad \gamma_\phi = A\, g_4^2 \,, \quad \beta_2 = -2g_2 + B\, g_2 g_4 \,, \quad \beta_4 = C\, g_4^2$

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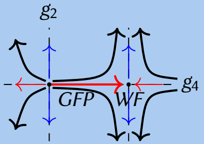
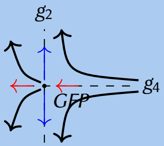
- Wilson Fisher $(g_4^\star, g_2^\star \;|\gamma_\phi^\star) = (\frac{1}{C}\epsilon, 0\;|\frac{A}{C^2}\epsilon^2)$, one relevant $\theta_{\phi^2} = 2 - \frac{B}{C}\epsilon$ and one irrelevant $\theta_{\phi^4} = -\epsilon$,



$d = 4 \; (\Delta_\phi = 1) \qquad \gamma_\phi = A\, g_4^2\,, \;\; \beta_2 = -2g_2 + B\, g_2 g_4\,, \;\; \beta_4 = C\, g_4^2$

Gaußian $(g_4^\star, g_2^\star \;|\gamma_\phi^*) = (0, 0\;|0)$, one relevant direction $\theta_{\phi^2} = 2$ but g_4 marginal – solve for $g(\Lambda)$

$$g_4(\Lambda) = \frac{g_4(\mu)}{1 - C g_4(\mu) \ln(\frac{\Lambda}{\mu})}\,, \qquad \Lambda_{\text{break down}} = \mu e^{\frac{1}{C\, g_4(\mu)}} > \mu$$



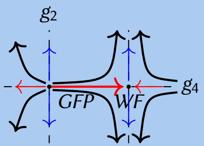
- $g_4 > 0$ trivial (free) infrared theory, no ultraviolet complete trajectory (Landau pole)
- $g_4 < 0$ ultraviolet complete (asymptotically free)

Fixed points in the ϕ^4 model

$$O(g^2): \qquad \gamma_\phi = A\, g_4^2\,, \qquad \beta_2 = -(d-2\Delta_\phi)\, g_2 + B\, g_2\, g_4\,, \qquad \beta_4 = -(d-4\Delta_\phi)\, g_4 + C\, g_4^2$$

$d = 4-\epsilon$ $(\Delta_\phi = 1-\frac{\epsilon}{2})$ $\qquad \gamma_\phi = A\, g_4^2\,,\quad \beta_2 = -2g_2 + B\, g_2\, g_4\,,\quad \beta_4 = -\epsilon g_4 + C\, g_4^2$

- Gaußian $(g_4^\star, g_2^\star\ |\gamma_\phi^\star) = (0, 0\ |0)$, two relevant directions $\theta_{\phi^4} = \epsilon$, $\theta_{\phi^2} = 2$
- Wilson Fisher $(g_4^\star, g_2^\star\ |\gamma_\phi^\star) = (\frac{1}{C}\epsilon, 0\ |\frac{A}{C^2}\epsilon^2)$, one relevant $\theta_{\phi^2} = 2 - \frac{B}{C}\epsilon$ and one irrelevant $\theta_{\phi^4} = -\epsilon$,



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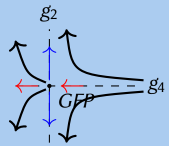
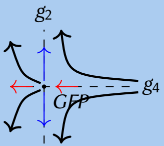


- $g_4 > 0$ trivial (free) infrared theory, no ultraviolet complete trajectory (Landau pole)
- $g_4 < 0$ ultraviolet complete (asymptotically free) but unstable – the “wrong sign ϕ^4 model”

The universality of the Landau pole

$$g_4(\Lambda) = \frac{g_4(\mu)}{1 - C g_4(\mu) \ln(\frac{\Lambda}{\mu})} \,, \qquad \Lambda_{\text{break down}} = \mu e^{\frac{1}{C\, g_4(\mu)}} > \mu$$



For a scalar field the Landau pole on the stable side $g_4 > 0$ is *unavoidable*:

$$Z_\Lambda = 1 - \lambda^2 \partial_{(-\Delta)} \,, \qquad g_4 = \mu^{-(d-4\Delta_\phi)} \frac{\lambda - \lambda^2}{Z_\Lambda^2}$$

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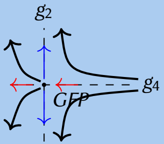

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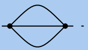
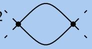
$Z_\Lambda = 1 - \lambda^2 \partial_{(-\Delta)}$  , $g_4 = \mu^{-(d-4\Delta_\phi)} \dfrac{\lambda - \lambda^2 \cdot (\text{diagram})}{Z_\Lambda^2}$



Then again, maybe not!

Tensors and invariants

Order 3 tensor $\psi'_{b_1 b_2 b_3} = O^{(1)}_{b_1 a_1} O^{(2)}_{b_2 a_2} O^{(3)}_{b_3 a_3} \psi_{a_1 a_2 a_3}$, invariant action

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Invariants contract indices in the same position:

- quadratic invariant $\sum_a \psi_{a_1 a_2 a_3} \psi_{a_1 a_2 a_3}$

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Invariants contract indices in the same position:

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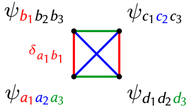
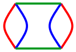
- quartic invariants:

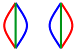
$$[\psi^4]_{tetrahedral} = \sum_{abcd} \psi_{a_1 a_2 a_3} \psi_{b_1 b_2 b_3} \psi_{c_1 c_2 c_3} \psi_{d_1 d_2 d_3} \left(\delta_{a_1 b_1} \delta_{c_1 d_1} \delta_{a_2 c_2} \delta_{b_2 d_2} \delta_{a_3 d_3} \delta_{b_3 c_3}\right)$$



$$[\psi^4]_{pillow} = \sum_{abcd} \psi_{a_1 a_2 a_3} \psi_{b_1 b_2 b_3} \quad \psi_{c_1 c_2 c_3} \psi_{d_1 d_2 d_3} (\delta_{a_1 b_1} \delta_{a_2 b_2}) \delta_{a_3 d_3} \delta_{b_3 c_3} (\delta_{c_1 d_1} \delta_{c_2 d_2})$$



$$[\psi^4]_{\text{double trace}} = \sum_{abcd} \psi_{a_1 a_2 a_3} \psi_{b_1 b_2 b_3} \quad \psi_{c_1 c_2 c_3} \psi_{d_1 d_2 d_3} (\delta_{a_1 b_1} \delta_{a_2 b_2} \delta_{a_3 b_3})(\delta_{c_1 d_1} \delta_{c_2 d_2} \delta_{c_3 d_3})$$



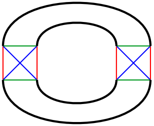
Partition function, correlations, $1/N$ and all that

Invariant action [Carrozza Tanasa '15, Giombi Klebanov Tarnopolsky '16 '17 '18]

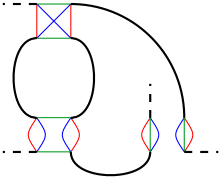
$$S = \int d^d x \left[\frac{1}{2} \psi_{a_1 a_2 a_3} (-\partial_\mu \partial^\mu) \psi_{a_1 a_2 a_3} + \frac{\lambda_t}{N^{3/2}} [\psi^4]_t + \frac{\lambda_p}{4N^2} [\psi^4]_p + \frac{\lambda_d}{4N^3} [\psi^4]_d \right]$$

Graphs – all indices are identified along propagators hence:

- free sum per "face", that is cycle of propagators and edges of one color
- pairwise identification of external indices



$$\sim \frac{N^6}{N^{3/2} N^{3/2}} = N^3$$



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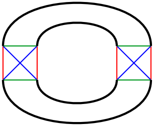

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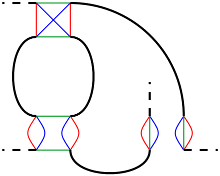

$$\sim \frac{N}{N^{3/2} N^2 N^3} [\phi^4]_t$$

$1/N$ expansion

$$\ln \int \mathcal{D}\psi \; e^{-S(\psi)} = N^3 F^{LO} + N^{\frac{5}{2}} F^{NLO} + O(N^2) \,, \qquad \langle [\phi^p]_{\text{invariant}} \rangle = \frac{1}{N^{LO}} \sum_{s \geq 0} N^{-\frac{s}{2}} G^{(s)}(x_i)$$

Flow of the tetrahedral coupling driven by the wave function at all orders in perturbation theory

$$g_t(\Lambda) = \mu^{-(d-4\Delta_\phi)} \frac{\lambda_t + \text{no radiative correction!}}{Z_\Lambda^2}$$

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Consistent second order truncation (suppress index t):

$$\gamma_\phi = \frac{1}{2} g^2 \,, \qquad \beta = -(d - 4\Delta_\phi - 4\gamma_\phi)\, g$$

$$\beta_p = -(d - 4\Delta_\phi - 4\gamma_\phi)\, g_p + 6g^2 + \frac{2}{3} g_p^2 - 4g^2 g_p$$

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$d = 4 - \epsilon$ dimensions [Giombi Klebanov Tarnopolsky '17]

$$\beta = -\epsilon g + 2g^3 , \qquad \beta_p = -\epsilon g_p + \left(6g^2 + \frac{2}{3} g_p^2\right) - 2g^2 g_p$$

$$\beta_d = -\epsilon g_d + \left(\frac{4}{3} g_p^2 + 4g_p g_d + 2g_d^2\right) - 2g^2 (4g_p + 5g_d) ,$$

$$g_\star = (\epsilon/2)^{1/2} , \qquad g_{p\star} = \pm \mathrm{i}\, 3(\epsilon/2)^{1/2} , \qquad g_{p\star} = \mp \mathrm{i}(3 \pm \sqrt{3})(\epsilon/2)^{1/2} ,$$

Wilson Fisher like fixed point with limit cycles.

Flow of the tetrahedral coupling driven by the wave function at all orders in perturbation theory

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$d = 4$ dimensions

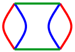
$$\beta = 2g^3 \,, \qquad \beta_p = 6g^2 + \frac{2}{3} g_p^2 - 2g^2 g_p \,, \qquad \beta_d = \frac{4}{3} g_p^2 + 4g_p g_d + 2g_d^2 - 2g^2(4g_p + 5g_d)$$

$$g(\Lambda)^2 = \frac{g(\mu)^2}{1 - 4g(\mu)^2 \ln(\frac{\Lambda}{\mu})}$$

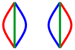
Gaußian fixed point is attractive on both sides – Landau pole (infrared trivial) for both signs of g

But not all the invariants have a definite sign!

$$[\psi^4]_{pillow} = \sum_{a_3 c_3} \left(\sum_{a_1 a_2 b_1 b_2} \psi_{a_1 a_2 a_3} \delta_{a_1 b_1} \delta_{a_2 b_2} \psi_{b_1 b_2 b_3} \right)^2 \geq 0$$



$$[\psi^4]_{\text{double trace}} = \left(\sum_{ab} \psi_{a_1 a_2 a_3} \delta_{a_1 b_1} \delta_{a_2 b_2} \delta_{a_3 b_3} \psi_{b_1 b_2 b_3} \right)^2 \geq 0$$



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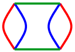

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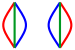
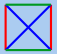


$$[\psi^4]_{tetrahedral} = \sum_{abcd} \psi_{a_1 a_2 a_3} \psi_{b_1 b_2 b_3} \psi_{c_1 c_2 c_3} \psi_{d_1 d_2 d_3} \left(\delta_{a_1 b_1} \delta_{c_1 d_1} \delta_{a_2 c_2} \delta_{b_2 d_2} \delta_{a_3 d_3} \delta_{b_3 c_3} \right) \quad ?$$



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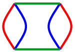

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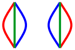

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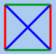

For ψ a discrete Fourier transform of a Wigner $3j$ symbol, $[\psi^4]_{tetrahedral}$ is the $6j$ symbol which changes sign with j

As $[\phi^4]_p, [\phi^4]_d \geq 0$ but $[\phi^4]_t$ can be positive or negative, stability guaranteed only if

- $g \to \imath g$ like in the Lee-Yang $\imath\phi^3$ model
- $g_1, g_2 > 0$ with $g_1 = g_p/3$, $g_2 = g_p + g_d$

An asymptotically free model

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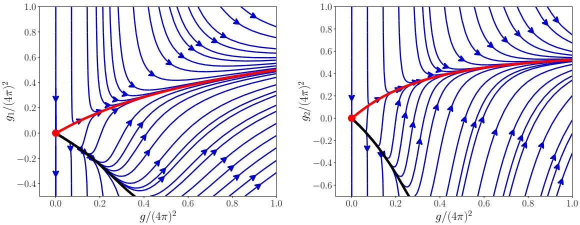
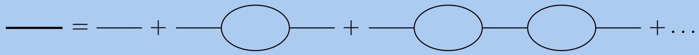
$$\gamma_\phi = -\frac{1}{2}g^2 \;, \qquad \beta = -2g^3 \;, \qquad \beta_1 = 2(g_1^2 - g^2 + g_1 g^2) \;, \qquad \beta_2 = 2(g_2^2 - 3g^2 + 5g_2 g^2)$$

$$g(\Lambda)^2 = \frac{g(\mu)^2}{1+4g(\mu)^2\ln(\frac{\Lambda}{\mu})} \;, \qquad Z_\Lambda = \left[1+4g(\mu)^2\ln(\frac{\Lambda}{\mu})\right]^{\frac{1}{4}} Z_\mu \;, \qquad \Lambda_{\rm QCD} = \mu e^{-\frac{1}{4g^2(\mu)}} < \mu$$



The red separatrix is asymptotically free and stable

Schwinger Dyson equation (self energy Σ, bare propagator C)



$$\langle\phi\phi\rangle_{1\to 2} \equiv G = C + C\Sigma C + C\Sigma C\Sigma C + \cdots = \frac{1}{C^{-1} - \Sigma}$$

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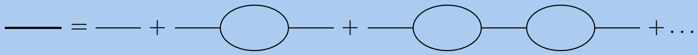
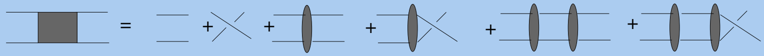


$$\langle\phi\phi\rangle_{1\to 2} \equiv G = C + C\Sigma C + C\Sigma C\Sigma C + \cdots = \frac{1}{C^{-1} - \Sigma}$$

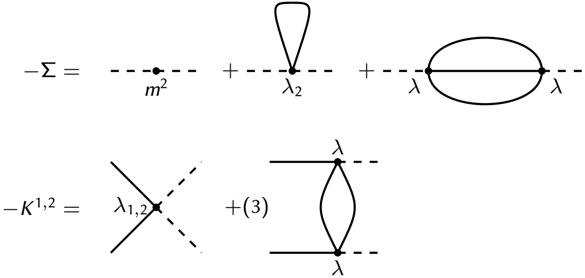
Four point Dyson equations (four point kernel $K = GG\frac{\delta\Sigma}{\delta G}$):



$$\langle\phi\phi\phi\phi\rangle_{12\to 34} = 2GG + 2KGG + 2K^2GG \cdots = 2\frac{1}{1-K}GG$$

Large N kernels

$$\Sigma_{xy} = -m^2\delta_{xy} - \lambda_2 G_{xx}\delta_{xy} - \lambda^2 G_{xy}^3 \qquad K^{1,2}_{xy;uv} = -\lambda_{1,2} G_{xu}G_{yv}\delta_{uv} - (3)\lambda^2 G_{xu}G_{yv}G_{uv}^2$$



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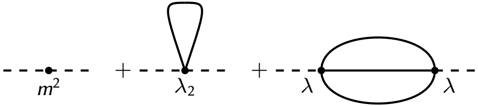
Self consistent equations for correlation functions in the large N limit

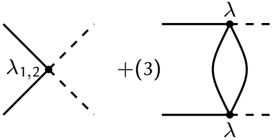
$$(G^{-1})_{xy} = -\Delta - \Sigma_{xy} = -\Delta + m^2\delta_{xy} + \lambda_2 G_{xx}\delta_{xy} + \lambda^2 G^3_{xy} \ ,$$

$$\Gamma^{1,2}_{xy;zt} = \lambda_{1,2}\delta_{xz}\delta_{zt}\delta_{zt} + (3)\lambda^2 G^2_{zt} + \int_{x'y'} \Gamma^{1,2}_{xy;x'y'} K^{1,2}_{x'y',zt}$$

$-\Sigma =$



$-K^{1,2} =$



Non perturbative flow

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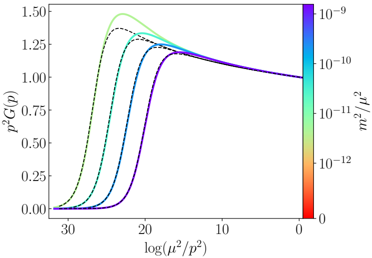

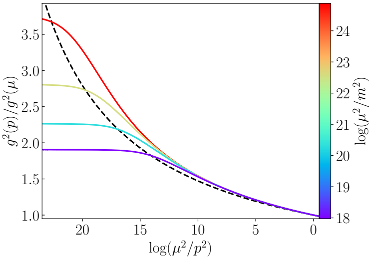

(in picture – p momentum, μ renormalization scale, m^2 renormalized mass)

Coupling to gravity

$$S = \int d^d x \sqrt{g+h} \left[\frac{1}{2} \psi_{a_1 a_2 a_3} (-\partial^\nu (g_{\mu\nu} + h_{\mu\nu}) \partial^\mu) \psi_{a_1 a_2 a_3} + \frac{\lambda_t}{N^{3/2}} [\psi^4]_t + \frac{\lambda_p}{4N^2} [\psi^4]_p + \frac{\lambda_d}{4N^3} [\psi^4]_d \right] + \frac{1}{2G_{\text{bare}}} (h_{\mu\nu} \cdot Q \cdot h^{\mu\nu})$$

$$\beta = -f_\lambda g - 2g^3 \;, \quad \beta_1 = -f_\lambda g_1 + 2(g_1^2 - g^2 + g_1 g^2) \;, \quad \beta_2 = -f_\lambda g_2 + 2(g_2^2 - 3g^2 + 5g_2 g^2)$$

- f_λ comes from one loop wave function with graviton + graviton self loop on $[\phi^4]$
- f_λ depends only on the gravitational couplings (same f_λ for all the quadratic couplings)
- $f_\lambda \sim G$ the dimensionless Newton constant hence $f_\lambda \to 0$ in the infrared as $G \sim \Lambda^2 / M^2_{\text{Planck}}$
- $f_\lambda < 0$ (need a student for this one, see Zois Gyftopoulos poster)

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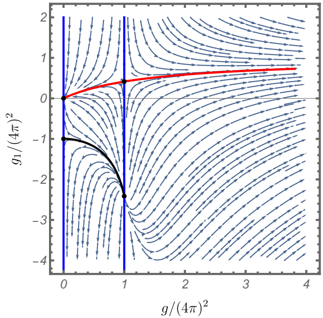
If gravity is asymptotically safe then in the ultraviolet $f_\lambda = f^\star_\lambda$, fixed point value

Reversed Wilson Fisher– the quartic coupling g is

- irrelevant at GFP
- relevant at $g_\star$ on the red separatrix



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- and asymptotically safe and stable scalar model
  - solvable at large N
  - real quantum effective action at large N
- a candidate ultraviolet fixed point for the model coupled to asymptotically safe gravity

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To do list:

- solve for the four point functions at large N
- study what happens at Λ_{QCD}

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Does the model make sense in Lorentzian signature?