Dressing the functional renormalization group

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Be aware!
PIRGS in the area!

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Physics-informed gauge theories

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We use the physics-informed renormalisation group (PIRG) for the construction of gauge invariant renormalisation group flows. The respective effective action is a sum of a gauge invariant quantum part and the classical gauge fixing part which arranges for invertibility of the gauge field two-point function. Thus, the BRST transformations simply accommodate the gauge consistency of the gauge fixing sector, while the quantum part of the effective action is gauge and BRST invariant. We apply this physics-informed approach to Yang-Mills theory and gravity and show how the flowing gauge fields arrange for full gauge invariance. We also embed the background field approximation to the functional renormalisation group (fRG) in an exact gauge invariant PIRG flow. This allows us to discuss the dynamics of the correction terms, and the non-trivial ultraviolet or infrared relevant terms are elucidated within a one-loop approximation. The background field approximation of the latter is known for violating one-loop universality for specific regulators and we show how the present setup reinstates universality in a constructive way. Finally, we discuss the landscape of fRG flows in gauge theories through the lens of the novel PIRG approach as well as potential applications.

Introduction

- In computations in the continuum, gauge invariance is typically dealt with by gauge fixing;
- This “replaces” gauge invariance by BRST symmetry
- Preserving BRST invariance allows for an easy control of spurious dependences such as gauge-parameter dependence;
- However, introducing regulators typically deform BRST invariance thanks to the mass-like behavior of such terms;
- Such a deformation is encoded in the so-called modified Ward identities (mWI) and modified Slavnov-Taylor identity (mSTI);

- In order to avoid such complications, several different gauge-invariant flow equations were proposed along the history of the FRG;
- In fact, we shall argue that at least some of those gauge-invariant formulations can be (nearly) recovered from gauge-fixed flows by the use of dressed gauge fields;

Can we construct gauge-invariant mass-like terms in gauge theories?

- **There are topologically massive gauge theories; There is the Higgs mechanism,…**
- **But can we add an explicit “mass” term (as a typical regulator) that is compatible with gauge/BRST invariance?**

Warming up: Abelian gauge theories (Euclidean Path Integral)

$$Z[J] = \int \mathscr{D}A\, \mathrm{e}^{-S_{\mathrm{M}}[A] + \int \mathrm{d}^d x\, J_\mu(x) A_\mu(x)}$$

generating functional of correlation functions

$$S_{\mathrm{M}}[A] = \frac{1}{4} \int \mathrm{d}^4 x\ F_{\mu\nu} F_{\mu\nu}$$

Maxwell's Action

Maxwell's action is invariant under Abelian gauge transformations written as

$$A'_\mu = A_\mu - \partial_\mu \xi$$

Such an invariance spoils the construction of the propagator of the photon field

GAUGE FIXING!

Faddeev-Popov Gauge-Fixing Procedure

We choose the Landau gauge for concreteness

$$Z[J] = \int \mathscr{D}A\, \delta(\partial_\mu A_\mu)\, \det \mathscr{M}_{\mathrm{FP}}\, \mathrm{e}^{-S_{\mathrm{M}}[A] + \int \mathrm{d}^d x\, J_\mu(x) A_\mu(x)}$$

Faddeev-Popov Identity

Faddeev-Popov Operator: $\mathscr{M}_{\mathrm{FP}} = -\partial^2$ (field independent)

Introduction of the so-called FP ghosts and the LN field:

$$Z[J] = \int [\mathscr{D}\mu]_{\mathrm{FP}}\, \mathrm{e}^{-S[A,b,\bar{c},c] + S_{\mathrm{sources}}}$$

$$[\mathscr{D}\mu]_{\mathrm{FP}} = [\mathscr{D}A][\mathscr{D}b][\mathscr{D}\bar{c}][\mathscr{D}c]$$

$$S[A,b,\bar{c},c] = S_{\mathrm{M}}[A] + S_{\mathrm{FP}}[A,b,\bar{c},c]$$

$$S_{\mathrm{FP}}[A,b,\bar{c},c] = \int \mathrm{d}^4 x\ b\, \partial_\mu A_\mu + \int \mathrm{d}^4 x\ \bar{c}\partial^2 c$$

BRST Symmetry

An important outcome of the FP quantization: BRST symmetry

$$sA_\mu = -\partial_\mu c$$
$$sc = 0$$
$$s\bar{c} = b$$
$$sb = 0$$

$$s^2 = 0$$

$$S_{\text{FP}}[A, b, \bar{c}, c] = s \int \mathrm{d}^4x \; \bar{c}\, \partial_\mu A_\mu$$

FP action is written as a BRST variation: BRST exact

Formally the same

$$\delta A_\mu = -\partial_\mu \xi$$

Gauge transformation

$$sA_\mu = -\partial_\mu c$$

BRST transformation

Maxwell's Action is thus invariant under BRST transformations but it is not BRST exact: BRST closed

Introducing a mass term

Let us introduce to the gauge-fixed Maxwell's action a "standard" mass term:

$$S_{m^2} = \frac{m^2}{2}\int \mathrm{d}^4x \; A_\mu A_\mu$$

$$sS_{m^2} = -m^2 \int \mathrm{d}^4x \; (\partial_\mu c) A_\mu \overset{!}{=} 0$$

This is invariant *on-shell* in the Landau gauge
Explicit (but soft) BRST breaking

The mass parameter m^2 does not appear in a BRST-exact term: It can appear in physical correlation functions.

$$\Sigma = S_{\mathrm{M}} + S_{\mathrm{FP}} + S_{m^2}$$

$$\frac{\partial \Sigma}{\partial m^2} \neq s(\ldots)$$

The "miraculous" (on-shell) gauge-invariance of the mass term comes by due to the gauge-invariant nature of the transverse component of the Abelian gauge field:

$$A^{\mathrm{T}}_\mu = A_\mu - \frac{\partial_\mu}{\partial^2}\partial \cdot A$$

Gauge Invariant! ⟶ **It is clear that the (non-local) transverse component of the gauge field colapses to the gauge-field itself in the Landau gauge.**

Introducing a mass term

We can say that the transverse component of the gauge field is a *dressing* of the gauge field

One could write down the mass-like term directly with the transverse component of the gauge field:

$$S^{\mathrm{T}}_{m^2} = \frac{m^2}{2}\int \mathrm{d}^4x \; A^{\mathrm{T}}_{\mu} A^{\mathrm{T}}_{\mu} \quad \longmapsto \quad sS^{\mathrm{T}}_{m^2} = 0$$

BRST-invariance is attained off-shell. No Free Lunch: The "mass" term is non-local!

Nevertheless, such a non-locality is harmless in the following sense:

$$S^{\mathrm{T}}_{m^2} = S_{m^2} + \int \mathrm{d}^4x \, \mathscr{F}(A)\partial_\mu A_\mu \quad \longmapsto \quad b^{\mathrm{T}} = b + \mathscr{F}(A)$$

$$Z = \int [\mathscr{D}\mu]_{\mathrm{FP}} \, \mathrm{e}^{-\Sigma^{\mathrm{T}}[A,b,\bar{c},c]} \quad \longmapsto \quad Z = \int [\mathscr{D}\mu]_{\mathrm{FP}} \, \mathrm{e}^{-\Sigma[A,b^{\mathrm{T}},\bar{c},c]}$$

$$\langle \mathscr{O}(x_1)\ldots\mathscr{O}(x_n)\rangle_{\Sigma^{\mathrm{T}}} = \langle \mathscr{O}(x_1)\ldots\mathscr{O}(x_n)\rangle_{\Sigma}$$

Correlation functions are exactly the same both in the explicitly BRST-broken (but local) formulation and in the manifestly BRST-invariant (non-local) form.

Coarse-graining and the fate of BRST invariance

Our interest is to apply Functional Renormalization Group (FRG) techniques and hence we introduce quadratic regulators on the elementary fields:

$$Z_k[J] = \int [\mathscr{D}\mu]_{\rm FP}\, \mathrm{e}^{-S[A,b,\bar{c},c]-\Delta S_k^{(A)}-\Delta S_k^{(\bar{c}c)}+S_{\rm sources}}$$

$$\Delta S_k^{(A)} = \frac{1}{2}\int \mathrm{d}^4x\; A_\mu\, \mathscr{R}^{\mu\nu}_{k,(A)}(-\partial^2)\, A_\nu$$

$$\Delta S_k^{(\bar{c}c)} = \int \mathrm{d}^4x\; \bar{c}\, \mathscr{R}_{k,(\bar{c}c)}(-\partial^2)\, c$$

Clearly, the regulator terms break BRST invariance. However, we can apply the same logic as before. We could try to employ the following BRST-invariant regulator:

$$\Delta S_k^{(A^T)} = \frac{1}{2}\int \mathrm{d}^4x\; A^{\rm T}_\mu\, \mathscr{R}^{\mu\nu}_{k,(A)}(-\partial^2)\, A^{\rm T}_\nu$$

with

$$A^{\rm T}_\mu = A_\mu - \frac{\partial_\mu}{\partial^2}\partial\cdot A$$

dressed gauge field

$$\Delta S_k^{(A^T)} = \Delta S_k^{(A)} + \int \mathrm{d}^4x\; \mathscr{F}_k(A)\, \partial\cdot A$$

Collecting the ghost terms:

$$S_{\rm FP}[0,0,\bar{c},c] + \Delta S_k^{(\bar{c}c)} = \int \mathrm{d}^4x\; \bar{c}\left(-\partial^2 + \mathscr{R}_{k,(\bar{c}c)}(-\partial^2)\right) c$$

Gauge-invariant coarse-graining

We now employ the Landau gauge. As a first step we integrate out the FP ghosts and replace the gauge-field regulator by the (dressed) gauge-invariant regulator

$$Z_k = \int [\mathscr{D}\mu]_{\rm FP} \left(\det \mathscr{M}_{\rm FP,k} \right) \mathrm{e}^{-S[A,b,\bar{c},c] - \Delta S_k^{(A^T)} + \int \mathrm{d}^4x \, \mathscr{F}_k(A)\, \partial\cdot A}$$

Ghost sector:

$$\det \mathscr{M}_{\rm FP,k} = \det \left(\partial^2 + \mathscr{R}_{\rm k,(\bar{c}c)}(-\partial^2) \right) = \exp\left[\mathrm{Tr}\, \ln\left(\partial^2 + \mathscr{R}_{k,(\bar{c}c)}(-\partial^2) \right) \right]$$

From the gauge-fixing term:

$$\int \mathrm{d}^4x \; b\, \partial_\mu A_\mu \longmapsto \int \mathrm{d}^4x \; b\, \partial_\mu A_\mu - \int \mathrm{d}^4x \; \mathscr{F}_k(A)\, \partial \cdot A$$

Consequently

$$\int \mathrm{d}^4x \; b^{\rm T}\, \partial_\mu A_\mu \longmapsto b^{\rm T} = b - \mathscr{F}_k(A)$$

trivial Jacobian

Gauge-invariant coarse-graining

Integrating out the redefined LN field:

$$Z_k[J] = \int [\mathscr{D}A]\, \delta(\partial \cdot A)\, \mathrm{e}^{-S_{\mathrm{M}}[A]-\Delta S_k^{(A^T)}+S_{\text{sources}}}$$

Ghost sector decouples - field-independent.

In the Landau gauge: $A_\mu^{\mathrm{T}} \to A_\mu$

In this sense, in the Landau gauge condition, the gauge field can be replaced by a gauge-invariant field with no extra cost. Hence, the regulator can be written in terms of gauge-invariant fields.

Thus, a "gauge-invariant" coarse-grained partition function is obtained from the gauge-fixed one.

It was key to use a dressing for the gauge field compatible with the gauge choice, i.e., on-shell, the dressed gauge field collapses to the gauge field itself.

Non-Abelian gauge theories

Let us consider now

$$Z[J] = \int \mathscr{D}A \, \mathrm{e}^{-S_{\mathrm{YM}}[A]}$$

Euclidean path integral

$$S_{\mathrm{YM}}[A] = \frac{1}{4}\int \mathrm{d}^4x \; F^a_{\mu\nu}F^a_{\mu\nu}$$

Yang-Mills Action

Gauge transformation

$$A'_\mu = U^\dagger A_\mu U + \frac{i}{g}U^\dagger \partial_\mu U$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$U \in SU(N) \qquad A_\mu = A^a_\mu T^a$$

$$[T^a, T^b] = i f^{abc} T^c$$

The transverse component of the gauge field is *not* gauge invariant. How to obtain the analogue to the “transverse component” in the non-Abelian case?

Choose your dressing: Gauge-field wardrobe

[ADP, Peruzzo, To appear]

Let us define the composite field $A^h_\mu \equiv A^{h,a}_\mu T^a$ by the introduction of the auxiliary field ξ

$$A^h_\mu = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$$

$$h = e^{ig\xi^a T^a}$$

Gauge transformations

$$A^U_\mu = U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U$$

$$h^U = U^\dagger h$$

$\longrightarrow$ $$(A^h)^U_\mu = A^h_\mu$$

The composite field A^h_μ is gauge invariant. Next to that, we can impose a gauge condition for A^h_μ

$\longrightarrow$ $$F[A^h_\mu] = 0$$

This is also gauge invariant!

$$A^h_\mu = A^h_\mu(A, \xi)$$ $\longrightarrow$ $$\xi = \mathcal{G}(A)$$

Imposing the gauge condition

$$A^h_\mu = A^h_\mu(A, \xi) = A^h_\mu(A, \mathcal{G}(A)) \equiv A^h_\mu(A)$$

Thus, the gauge-invariant composite field A^h_μ is expressed solely in terms of A_μ

General idea

$$A_\mu^h = A_\mu - \partial_\mu \xi + g\mathcal{Q}_\mu(g, A, \xi) \qquad F[A_\mu] = \partial_\mu A_\mu + g\mathcal{P}(A) = 0$$

$$F[A_\mu - \partial_\mu \xi + g\mathcal{Q}_\mu(g, A, \xi)] = 0 \qquad \xi = \frac{1}{\partial^2} F[A] + \frac{g}{\partial^2}\mathcal{K}(A, \xi)$$

Solving the implicit equation for ξ introduces the functional $F[A]$

IF the gauge field A_μ satisfies $F[A] = 0$ the solution for ξ becomes TRIVIAL!

$$A_\mu^h = A_\mu - \frac{\partial_\mu}{\partial^2} F[A] + g(\partial_\mu \mathcal{O}(A, g))F[A] \longrightarrow A_\mu^h \stackrel{!}{=} A_\mu$$

A_μ^h is gauge invariant (and tremendously non-local)

Dressing implies gauge invariance but there is no direct physical meaning
Dressed fields are not necessarily "physical"

Example: Landau Dressing

$$F[A_\mu] = \partial_\mu A_\mu = 0$$

The explicit expression for A^h_μ as a function of the gauge field is

$$A^h_\mu = A_\mu - \frac{\partial_\mu}{\partial^2}\partial \cdot A + ig\left[A_\mu, \frac{1}{\partial^2}\partial \cdot A\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial \cdot A, \partial_\mu \frac{1}{\partial^2}\partial \cdot A\right] + ig\frac{\partial_\mu}{\partial^2}\left[\frac{\partial_\nu}{\partial^2}\partial \cdot A, A_\nu\right] + \frac{ig}{2}\frac{\partial_\mu}{\partial^2}\left[\frac{\partial \cdot A}{\partial^2}, \partial \cdot A\right] + \mathcal{O}(A^3)$$

- The (Landau) dressed field A^h_μ is gauge invariant;
- It is transverse, $\partial_\mu A^h_\mu = 0$; (aka $F[A^h_\mu] = 0$)
- It reduces to the transverse component of the gauge field in the Abelian limit;
- Apart from the first term (that is the gauge field itself), all terms contain at least one factor of $\partial \cdot A$.

The above expression can be obtained by minimizing the following functional:

$$f_A[U] = \mathrm{Tr}\int \mathrm{d}^4x \; A^U_\mu A^U_\mu$$

Non-Abelian gauge theories: Gauge Fixing

Let us consider now

Gauge-fixed path integral (Landau gauge)

$$Z[J] = \int \mathscr{D}A\, \delta(\partial_\mu A^a_\mu)\, \det \mathscr{M}_{\rm FP}\, \mathrm{e}^{-S_{\rm YM}[A] + \int \mathrm{d}^d x\, J^a_\mu(x) A^a_\mu(x)}$$

$$\mathscr{M}^{ab}_{FP} = -\partial_\mu D^{ab}_\mu$$

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc} A^c_\mu$$

FP operator is field dependent

$$Z[J] = \int [\mathscr{D}\mu]_{\rm FP}\, \mathrm{e}^{-S[A,b,\bar{c},c] + S_{\rm sources}}$$

$$S[A,b,\bar{c},c] = S_{\rm YM} + \int \mathrm{d}^4 x \left(b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

BRST Transformations

$$s A^a_\mu = -D^{ab}_\mu c^b$$

$$s c^a = \frac{g}{2} f^{abc} c^b c^c$$

$$s \bar{c}^a = b^a$$

$$s b^a = 0$$

$$s^2 = 0$$

$$s A^h_\mu = 0$$

Non-Abelian gauge theories: Gauge Fixing

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$$Z[J] = \int [\mathscr{D}\mu]_{\rm FP}\, e^{-S[A,b,\bar{c},c] + S_{\rm sources}}$$

$$S[A,b,\bar{c},c] = S_{\rm YM} + \int d^4x \left(b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

BRST Transformations

$$sA^a_\mu = -D^{ab}_\mu c^b$$

$$s^2 = 0$$

$$sA^h_\mu = 0$$

Can we introduce a gauge-invariant regulator for the gauge field?

Dressing the gauge field [ADP, To appear]

In the Landau gauge: $A_\mu^h \to A_\mu$

We can introduce the following gauge-invariant regulator:

$$\Delta S_k^{(A^h)} = \frac{1}{2}\int \mathrm{d}^4x \; A_\mu^{h,a}\, \mathscr{R}_{k,\mu\nu}^{ab}(-\partial^2)\, A_\nu^{h,b}$$

$$Z_k = \int \mathscr{D}A\, \delta(\partial_\mu A_\mu^a)\, \det \mathscr{M}_{\mathrm{FP}}(A)\, \mathrm{e}^{-S_{\mathrm{YM}}[A] - \Delta S_k^{(A^h)}}$$

The presence of the delta functional allows for the following replacement:

$$\det \mathscr{M}_{\mathrm{FP}}(A) \to \det \mathscr{M}_{\mathrm{FP}}(A^h) \longrightarrow$$

$$\det(-\delta^{ab}\partial^2 + g f^{abc} A_\mu^{h,c}\partial_\mu)$$

$$\downarrow$$

$$\det(\delta^{ab} P_k(-\partial^2) + g f^{abc} A_\mu^{h,c}\partial_\mu)$$

This looks very much with the logic of the gauge-invariant flow equation by C. Wetterich

Recovering the standard flow

In the Landau gauge:

$$\Delta S_k^{(A^h)} = \frac{1}{2}\int \mathrm{d}^4x \; A_\mu^{h,a}\, \mathscr{R}_{k,\mu\nu}^{ab}(-\partial^2)\, A_\nu^{h,b}$$

$$\Delta S_k^{(A^h)} = \Delta S_k^{(A)} + \int \mathrm{d}^4x \; \mathscr{F}^a(A)\,(\partial \cdot A^a)$$

The presence of the delta functional allows for: $\Delta S_k^{(A^h)} \to \Delta S_k^{(A)}$

$$\det \mathscr{M}_{\mathrm{FP,k}}(A^h) \to \det \mathscr{M}_{\mathrm{FP,k}}(A) = \det(-\delta^{ab}\partial^2 + g f^{abc} A_\mu^c \partial_\mu + \delta^{ab}\mathscr{R}_k(-\partial^2))$$

Lifting the regularized FP operator into the Boltzmann weight:

$$Z_k[J] = \int \mathscr{D}A\mathscr{D}\bar{c}\mathscr{D}c \; \mathrm{e}^{-S[A,\bar{c},c]-\Delta S_k^{(A)}-\Delta S_k^{(\bar{c}c)}+S_{\mathrm{sources}}}$$

back to the "standard" construction

Remark: If a different gauge is employed, the gauge-invariant regulator is not quadratic on the fields. However, one can choose a different dressing using the prescription presented before.

Recovering the standard flow (beyond Landau gauge)

Remark: In view of the different type of dressings the same reasoning could be applied for different gauge conditions, provided that one chooses compatible dressings and gauge-fixing conditions.

$$\Delta S_k^{(A^h)} = \Delta S_k^{(A)} + \int d^4x \; \tilde{\mathscr{F}}^a(A)\, F^a[A]$$

Once again, the "non-local garbage" can be absorbed in the Nakanishi-Lautrup field

$$\int d^4x \; b^{h,a} F^a[A_\mu] \equiv \int d^4x \; b^a \, F^a[A_\mu] + \int d^4x \; \mathscr{F}_k^a(A) F^a[A_\mu]$$

The associated (dressed) Faddeev-Popov operator can be regularized as before (in the Landau gauge)

$$\det \mathscr{M}_k(A^h) \to \det \mathscr{M}_k(A)$$

(due to the delta-functional enforcing the gauge condition)

In this formal sense: nothing special about the Landau gauge (but it is certainly a very convenient gauge)

Remark: Background Field Method

$$A_\mu^a = \bar{A}_\mu^a + a_\mu^a$$

$$\bar{D}_\mu^{ab} a_\mu^b = 0$$

$$(a^h)_\mu + \bar{A}_\mu = h^\dagger(\bar{A}_\mu + a_\mu)h + h^\dagger \partial_\mu h$$

$$(a^h)_\mu = a_\mu - \bar{D}_\mu \xi + ig[a_\mu, \xi] + H_\mu(\bar{A}, a, \xi)$$

Imposing $\bar{D}_\mu (a^h)_\mu = 0$

$$\xi = \frac{1}{\bar{D}^2}\bar{D}_\mu a_\mu + \frac{ig}{\bar{D}^2}\bar{D}_\mu [a_\mu, \xi] + \frac{ig}{\bar{D}^2}\bar{D}_\mu H_\mu(\bar{A}, a, \xi)$$

$$(a^h)_\mu = a_\mu - \bar{D}_\mu \frac{1}{\bar{D}^2}\bar{D}_\nu a_\nu + ig\left[a_\mu, \frac{1}{\bar{D}^2}\bar{D}_\nu a_\nu\right] + \mathscr{P}_\mu(\bar{A}, a, g)\bar{D}_\nu a_\nu$$

$$\Delta S_k^{(a^h)} = \frac{1}{2}\int \mathrm{d}^4 x \; a_\mu^{h,a} \, \mathscr{R}_{k,\mu\nu}^{ab}(-\partial^2) \, a_\nu^{h,b}$$

Localization

The dressed fields, when written in terms of the gauge field, are typically non-local. However, "as usual" we can localize them by introducing suitable auxiliary fields

We promote ξ to a field. Hence, the dressed field is written in the following non-polynomial form

$$A^h_\mu = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$$

$$h = \mathrm{e}^{ig\xi^a T^a}$$

$$\tilde{s} A^{h,a}_\mu = -D^{ab}_\mu(A^h)\,\eta^b$$

$$\tilde{s}\eta^a = \frac{g}{2} f^{abc}\eta^b\eta^c$$

$$\tilde{s}\bar{\eta}^a = \tau^a$$

$$\tilde{s}\tau^a = 0$$

We can introduce a "mass-like" term using the dressed (local) fields

$$S^{\mathrm{loc}}_{m^2} = \frac{m^2}{2}\int \mathrm{d}^4x\, A^{h,a}_\mu A^{h,a}_\mu$$

The implementation of the gauge-fixing condition $F[A^h_\mu]$ is achieved as follows:

$$S_{\mathrm{aux}} = \tilde{s}\int d^4x\, \bar{\eta}^a F[A^h_\mu]$$

$$S_{\mathrm{aux}} = \int d^4x \left(\tau^a F[A^h_\mu] - \bar{\eta}^a \tilde{s} F[A^h]\right)$$

$$\tilde{s} S_{\mathrm{aux}} = 0$$

Localization

$$\Sigma = S_{\mathrm{YM}} + S_{\mathrm{FP}} + S^{\mathrm{loc}}_{m^2} + S_{\mathrm{aux}}$$

Local (but non-polynomial) "massive" gauge-fixed YM action

This action is invariant under

$$sA^a_\mu = -D^{ab}_\mu c^b \qquad s\eta^a = 0$$

$$sc^a = \frac{g}{2} f^{abc} c^b c^c \qquad s\bar{\eta}^a = 0$$

$$s\tau^a = 0$$

$$s\bar{c}^a = b^a \qquad s\xi^a = g^{ab}(\xi)\, c^b$$

$$sb^a = 0 \qquad sA^{h,a}_\mu = 0$$

$$g^{ab}(\xi) = -\delta^{ab} + \frac{g}{2} f^{abc}\xi^c - \frac{g^2}{12} f^{ade} f^{dbm} \xi^m \xi^e + \mathcal{O}(\xi^3)$$

$$s^2 = 0$$

What about quantum gravity? [ADP, To appear]

Can we define a dressing for the metric fluctuations? YES!

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$x^\alpha = x'^\alpha + f^\alpha(x')$$

We define the dressed-like metric as

$$\mathbb{G}_{\mu\nu}(x) = \mathbb{T}^\alpha{}_\mu \mathbb{T}^\beta{}_\nu \, g_{\alpha\beta}(x)$$

$$\mathbb{T}'^\alpha{}_\mu(x') = \frac{\partial x'^\alpha}{\partial x^\lambda}\mathbb{T}^\lambda{}_\mu(x)$$

Hence,

$$\mathbb{G}'_{\mu\nu}(x') = \mathbb{T}'^\alpha{}_\mu \mathbb{T}'^\beta{}_\nu \, g'_{\alpha\beta}(x') = \frac{\partial x'^\alpha}{\partial x^\sigma}\frac{\partial x'^\beta}{\partial x^\rho}\frac{\partial x^\lambda}{\partial x'^\alpha}\frac{\partial x^\kappa}{\partial x'^\beta}\mathbb{T}^\sigma{}_\mu \mathbb{T}^\rho{}_\nu g_{\lambda k} = \mathbb{T}^\sigma{}_\mu \mathbb{T}^\rho{}_\nu \, g_{\sigma\rho}(x) = \mathbb{G}_{\mu\nu}(x)$$

What about quantum gravity?

$$\hat{h}_{\mu\nu}(x) = h_{\mu\nu}(x) + \bar{\nabla}_\mu \xi_\nu(x) + \bar{\nabla}_\nu \xi_\mu(x) + \mathbb{S}_{\mu\nu}(\bar{g}, h, \xi)$$

$$F_\mu[\bar{g}, \hat{h}_{\mu\nu}] = 0 \quad \textbf{e.g.} \quad F[\bar{g}, \hat{h}] = \bar{\nabla}^\mu \hat{h}_{\mu\nu} - \frac{1}{2}\bar{\nabla}_\nu \hat{h}$$

$$\xi_\mu = \left[\mathbb{M}^{-1}\right]_\mu^{\ \nu} F_\nu[h] + (\Xi(\bar{g}, h)\cdot F[h])_\mu \qquad \left[\mathbb{M}\right]_\mu^{\ \nu} = -\delta_\mu^\nu \bar{\nabla}^2 - \bar{R}_\mu^\nu$$

$$F[\bar{g}, \hat{h}] = \bar{\nabla}^\mu \hat{h}_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\nu \hat{h}$$

Choose your favorite gauge parameter, get a glass (bottle) of wine and go ahead…

The same logic with the “dressed regulator” can be applied and a gauge-invariant flow coarse-grained partition function can be written.

Curiosity…

The dressing for $\beta = 1$ can be obtained by the minimization of the following functional,

Minimizing functional

$$f_h[\epsilon] = \frac{1}{2}\int \mathrm{d}^4x\sqrt{\bar{g}}\;\bar{g}^{\mu\nu}h^{\epsilon}_{\mu\nu}$$

Dressed field: $\hat{h}_{\mu\nu}$ It is possible to show that: $\hat{h}_{\mu\nu} \to h_{\mu\nu}$ for $\alpha = 0 \quad \beta = 1$

With:

$$F_\mu[\bar{g};h] = \bar{\nabla}_\nu h^\nu{}_\mu - \frac{1+\beta}{4}\bar{\nabla}_\mu h$$

Conclusions

- Gauge-fixing 'seems to be' unavoidable in order to perform concrete computations;
- Clearly, choosing different gauges should not affect physical quantities;
- However, convenience is always a reasonable criterion for choosing a gauge;
- The Landau gauge in (non-)Abelian gauge theories is a very convenient choice;
- We have shown how to reconstruct a gauge-fixed flow from a gauge-invariant one in the Landau gauge by the use of dressed fields;
- We have seen that this procedure can be generalized to other gauges beyond the Landau choice;
- In fact such a procedure is rather algorithmic and can be applied also to background gauges and in quantum gravity;
- Of course, those different formulations for the same coarse-grained path integral can offer different approximation schemes and, certainly, different conceptual insights;

Thank You!

Physical (dressed) matter fields:

Let us write down the action for QED in the Landau gauge:

$$S_{\mathrm{QED}}[\Phi] = S_{\mathrm{M}}[A] + S_{\mathrm{FP}}[A, b, \bar{c}, c] + S_{\mathrm{D}}[\bar{\psi}, \psi, A]$$

$$S_{\mathrm{D}}[\bar{\psi}, \psi, A] = \int \mathrm{d}^4 x \left(\bar{\psi}\, \gamma_\mu D_\mu \psi - m\, \bar{\psi} \psi \right)$$

$$D_\mu = \partial_\mu - igA_\mu$$

We can define a gauge-invariant (dressed) field as follows:

$$\psi^h = \exp\left(-ig \frac{\partial \cdot A}{\partial^2} \right) \psi$$

$$\bar{\psi}^h = \bar{\psi} \exp\left(ig \frac{\partial \cdot A}{\partial^2} \right)$$

Gauge-invariant dressed fermions

Dressed matter fields:

Gauge-invariant regulator

$$\Delta S_k^{(\bar\psi^h\psi^h)} = \int \mathrm{d}^4x\ \bar\psi^h\,\mathscr{R}_{k,(\bar\psi\psi)}(-\partial^2)\,\psi^h \quad \longrightarrow \quad \Delta S_k^{(\bar\psi\psi)} = \int \mathrm{d}^4x\ \bar\psi\,\mathscr{R}_{k,(\bar\psi\psi)}(-\partial^2)\,\psi$$

In the Landau gauge

Once again, in the Landau gauge, the gauge-invariant (non-quadratic) regulator collapses into a quadratic expression: "physical gauge"

This is some sort of miraculous property of the Landau gauge! (Is it?)

In the Landau gauge, we can write the coarse-grained path integral with dressed fields which engender a gauge-invariant meaning to it or, conversely, a "physical" meaning to the Landau gauge.

One can map the standard regularized path integral to the gauge-invariant regularized path integral by a change of variables in the Landau gauge.

Is this a particular property of the Landau gauge? Can we generalize this type of reasoning to the non-Abelian case? What about gravity?