# Baryon number and other global symmetries in field theories of quantum gravity

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# Symmetries ...

#### ... are an impotant part of fundamental theories (physics)

Allows one to organise 'zoo' of particles (excitations) into multiplets
 elementary particles
 composite particles
 quasiparticles





q = -1 q = 0

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- Determines selection rules, forbids certain processes  $\Rightarrow$  stability
- (Folk) theorem: In quantum gravity, any continuous symmetry must be gauge Banks/Dixon '88; Giddings/Strominger '88; Kallosh *et al.* '95; Arkani-Hamed *et al.* '07; Banks/Seiberg '11; Harlow/Ooguri 19, 21; ...

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• ...most fundamental symmetries are gauged in nature e.g., charge conservation  $\leftrightarrow$  U(1) SM: U(1)  $\times$  SU(2)  $\times$  SU(3) is gauged

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Decay rate suppressed by powers of Planck mass
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*Q:* Are there any continuous global symmetries where QG-induced breaking leads to a particle being (observably) unstable?

- Decay rate suppressed by powers of Planck mass
   ⇒ particle has to come with strong experimental lower bounds on lifetime
- Example: proton decay  $p^+ 
  ightarrow \pi^0 e^+ \gamma \gamma$ 
  - $\Rightarrow$  forbidden by baryon number conservation, symmetry  $U(1)_{\text{B}}$  only global  $\Rightarrow$  potential candidate

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- Experimental non-observation leads to lower bounds for proton lifetime  $\tau_p$
- Current estimate:  $au_p\gtrsim 10^{34}\,{
  m yrs}$  Super-Kamiokande '17

#### Proton stability ...

... has already been a (serious) constraint on other deep-UV physics, e.g., GUTs









cf., e.g., Manohar '18  

$$\tau_p \approx 16\pi M_p^{-1} \left( G_{4\mathsf{F}}^{qqql}(k = M_p) \right)^{-2}$$

$$\mathsf{dim'less}$$

$$G_{4\mathsf{F}}(k) = \bar{G}_{4\mathsf{F}}(k)k^2$$





$$\tau_p \sim G_{\rm 4F}^{qqql}(M_X) \frac{M_X^4}{M_p^5}$$



#### **Remarks**:

- IR measurement ( $M_p \approx 1 \, {
  m GeV}$ ) constrains deep UV ( $M_{
  m exp} pprox 2 imes 10^{16} \, {
  m GeV}$ ).
- $M_{\text{GUT}} \approx M_{\text{exp}}$  Caveat:  $G_{4\text{F}}^{qqql}(M_X) \approx 1$  ('naturalness')
- E.g., room for viable GUTs with  $M_X\equiv M_{\rm GUT}\sim M_{\rm exp}$  if  $G_{
  m 4F}^{qqql}(M_X)\ll 1$

#### Proton stability and new physics at high energies...



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## Proton stability and quantum gravity



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- Heuristic picture: virtual black holes adapted from: Barrow '87; Alsaleh et al. '17

estimated proton lifetime: Zel'dovich '76; Adams et al. '01; ...

$$au_p \sim M_p^{-1} \left(rac{M_{
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• Here: Explicitly test validity of (\*) within Asymptotically Safe Quantum Gravity (ASQG)

#### Key assumption:







#### \* Toy model for QFT(SM + metric):

$$\begin{split} S &= S_{\rm EH} + S_{\rm kin,F} + S_{\rm 4F} \\ S_{\rm EH} &= \frac{1}{16\pi G_{\rm N}} \int_x \sqrt{g} (-R + 2\Lambda_{\rm cc}) \\ S_{\rm kin,F} &= \int_x \sqrt{g} \bar{\psi} \mathrm{i} \nabla \psi \\ S_{\rm 4F} &= \bar{G}_{\rm 4F}^{ABCD} \int_x \sqrt{g} \Psi_A \Psi_B \Psi_C \Psi_D \qquad \Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix} \end{split}$$



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#### \* $\psi$ : contains all SM fermions $\Psi$ = Nambu–Gor'kov spinor

...Dirac fermions, right-handed neutrinos included; SU(2)<sub>L</sub> gauge coupling asymptotically free in ASQG

\* split 
$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

 $\label{eq:constraint} \begin{array}{l} \dots \text{ in general: eigenvalues of } -\Delta_{\tilde{\mathcal{S}}} \text{ defines notion of scale} \\ \dots \text{ often in practice (i.e., here): } \\ \tilde{g}_{\mu\nu} \to \delta_{\mu\nu} \Longrightarrow \\ \text{ momentum is} \\ \text{ 'good quantum number' after all } \dots \end{array}$ 

### (Pure) gravity sector

$$\begin{split} S &= S_{\rm EH} + S_{\rm kin,F} + S_{\rm 4F} \\ S_{\rm EH} &= \frac{1}{16\pi G_{\rm N}} \int_x \sqrt{g} (-R + 2\Lambda_{\rm cc}) \\ S_{\rm kin,F} &= \int_x \sqrt{g} \bar{\psi} i \nabla \psi \\ S_{\rm 4F} &= \bar{G}_{\rm 4F}^{ABCD} \int_x \sqrt{g} \Psi_A \Psi_B \Psi_C \Psi_D \qquad \Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix} \end{split}$$

- Fluctuations  $h_{\mu\nu}$  decompose into spin-2, 1, and 0 parts
- Landau–DeWitt gauge: only transverse traceless  $h_{\mu\nu}^{\perp}$  and conformal  $h = h_{\mu}^{\mu}$  modes propagate

$$\frac{h_{\mu\nu}^{\perp}}{p^2 - 2\Lambda_{\rm cc}} = \frac{32\pi G_{\rm N}}{p^2 - 2\Lambda_{\rm cc}} (\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu} + \dots)$$
$$\frac{h}{p^2 - 2\Lambda_{\rm cc}} = \frac{32\pi G_{\rm N}}{-\frac{3}{8}p^2 + \frac{1}{2}\Lambda_{\rm cc}}$$

#### Remark: (Pure) gravity sector – renormalisation

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• Use as 'backdrop' for fermions (i.e.: neglect backreaction of fermions on metric)

#### Fermions

- Propagator has standard form  $-----= \frac{p}{n^2}$
- Vertices coupling metric fluctuations with fermions from  $\nabla$  and  $\sqrt{g}$ ...keep only to  $\mathcal{O}((h_{\mu\nu})^2)$



$$S = S_{\rm EH} + S_{\rm kin,F} + S_{\rm 4F}$$

$$S_{\rm EH} = \frac{1}{16\pi G_{\rm N}} \int_x \sqrt{g} (-R + 2\Lambda_{\rm cc})$$

$$S_{\rm kin,F} = \int_x \sqrt{g} \bar{\psi} i \nabla \psi \qquad \Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}$$

$$S_{\rm 4F} = \bar{G}_{\rm 4F}^{ABCD} \int_x \sqrt{g} \Psi_A \Psi_B \Psi_C \Psi_D$$

•  $G_{4F}^{ABCD}$ : Most general 4-Fermi interaction; proton decay ~  $G_{4F}^{qqql}$  cf.: Grzadkowski *et al.* '10  $\mathcal{O}(\bar{G}_{4F}^{ABCD})$ 

## Computational framework

$$\frac{\partial \Gamma_k[\Phi]}{\partial \ln k} = \frac{1}{2} \operatorname{STr} \left[ \left( \frac{\delta^2 \Gamma_k[\Phi]}{\delta \Phi \delta \Phi^\top} + R_k[\Phi] \right)^{-1} \frac{\partial R_k[\Phi]}{\partial \ln k} \right] \\ \xrightarrow{\Phi = (h_{dv}^\perp, h, \psi, \overline{\psi}^c)^\top; k - \operatorname{RG scale}; R - \operatorname{regulator}} = \mathbf{O}$$

cf., e.g.: Berges *et al.* Phys. Rep. '02; Metzner *et al.* Rev. Mod. Phys. '12; Dupuis *et al.* Phys. Rept. '21; and refs. therein

#### Functional renormalization group (FRG), general version

- $\Gamma$  1PI effective action aka Legendre effective action, quantum effective action, ...
  - $\Gamma_k$  average 1PI effective action aka 'blocked' –"-
    - fluctuations above scale k 'integrated out'
- 1-loop exact in principle assuming self-consistent solution loop expansion – start with  $\Gamma = S$  plus fixed-point iteration
- Ansatz for  $\Gamma_k$  defines approximation scheme
- Often: expand in canonical dimension (i.e., powers of ψ, h<sub>μν</sub>, ∂<sub>μ</sub>), keep least irrelvant terms justification: 'near perturbative' nature, cf. Codello/Percacci '06; Niedermaier '09, '10; Eichhorn *et al.* '18a,b; ...
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#### Functional renormalization group (FRG), 'quick and dirty' version

• Ansatz: 
$$\Gamma_k = S|_{h_{\mu\nu} \to \sqrt{Z_N} h_{\mu\nu}, G_N \to G_N(k), \Lambda_{cc} \to \Lambda_{cc}(k), \psi \to \sqrt{Z_{\psi}} \psi, G_F \to G_F(k)}$$

- Draw one-loop diagrams with vertices and propagators from before
- Replace couplings and propagators with 'dressed' versions
- Replace momentum integrals with 'threshold functions' diagram with n<sub>F</sub> internal fermion lines, n<sub>⊥</sub> spin-2 lines, n<sub>conf</sub> conformal mode lines ⇒ I<sub>n<sub>F</sub>,n<sub>⊥</sub>,n<sub>conf</sub>
  </sub>

e.g.:  

$$\int_{p}' \frac{32\pi G_{\rm N}}{-\frac{3}{8}p^{2} + \frac{1}{2}\Lambda_{\rm cc}} \mapsto I_{001} \sim \frac{(-6+\eta_{\rm N})g_{\rm N}}{(3-4\lambda_{\rm cc})^{2}}$$

$$\eta_{\rm N} = -k\partial_{k}\ln Z_{\rm N}, g_{\rm N} = G_{\rm N}k^{2}; \lambda_{\rm cc} = \Lambda_{\rm cc}/k^{2}$$

# Diagrammatics

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• **Result:** N.B.:  $\eta_{4F}$  in fact independent of precise index structure *ABCD*  $\leftrightarrow$  'gravity blind to internal indices'

$$k\partial_k G_{4\mathsf{F}}^{qqql}(k) = (2 + \eta_{4\mathsf{F}}) \, G_{4\mathsf{F}}^{qqql}(k) + \mathcal{O}((G_{4\mathsf{F}}^{qqql})^2)$$

• Explicitly: Litim regulator, Landau-DeWitt gauge, cf. Eichhorn/Gies '11

$$\begin{split} \eta_{4\mathsf{F}} &= 2g_{\mathsf{N}} \left[ -\frac{9(2\lambda-3)}{4\pi(3-4\lambda_{\mathsf{cc}})^2} + \frac{6(4\lambda_{\mathsf{cc}}-9)}{5\pi(3-4\lambda_{\mathsf{cc}})^2} + \frac{5}{4\pi(1-\lambda_{\mathsf{cc}})^2} + \frac{3}{2\pi(4\lambda_{\mathsf{cc}}-3)^2} \right] \\ &= \frac{29g_{\mathsf{N}}}{15\pi} + \frac{32g_{\mathsf{N}}\lambda_{\mathsf{cc}}}{9\pi} + \mathcal{O}(\lambda_{\mathsf{cc}}^2) \end{split}$$

## **Discussion I: General**

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$$k\partial_k G_{4\mathsf{F}}^{qqql}(k) = (2 + \eta_{4\mathsf{F}}) \, G_{4\mathsf{F}}^{qqql}(k) + \mathcal{O}((G_{4\mathsf{F}}^{qqql})^2)$$



• Generally:  $\eta_{4F} > 0$ ...assuming  $\lambda_{cc} > -9$  (pheno. relevant) 'metric fluctuations suppress proton decay'

• hence *a fortiori*: 
$$2 + \eta_{4F} > 0$$

⇒ naïvely 'unnatural'  $G_{4F}^{qqql}(M_{QG}) \ll 1$  is actually 'natural' if QFT(SM + metric) holds at  $M_{QG} < k < k_{UV}$  for  $k_{UV}$  large enough

N.B.: *Much* milder assumption than (eff.) AS! Caveats/assumptions:

\* Einstein–Hilbert truncation should remain good approximation

\* B -violation from UV completion is (at most) 'natural':  $G_{\rm 4F}^{qqql}(k_{\rm UV})\sim 1$ 

# Discussion II: AS and effective AS

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• Assumption: Running of  $g_N$ ,  $\lambda_{cc}$  negligible for  $M_{QG} < k < k_{UV}$  (= quasi-FP regime)

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Integrated flow:

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• Consider GUT with simple gauge group, coupling  $g_{GUT}$ 

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- Mixing of quarks and leptons  $\implies$  GUT gauge fluctuations induce  $G_{4F}^{qqql}$

 $k\partial_k G_{4\mathsf{F}}^{qqql} = 2G_{4\mathsf{F}}^{qqql} + C_0 g_{\mathrm{GUT}}^4$ 

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$$k\partial_k G^{qqql}_{4\mathsf{F}} = 2G^{qqql}_{4\mathsf{F}} + C_0 g^4_{\mathrm{GUT}}$$

• Assume  $k\partial_k g_{\text{GUT}} = b_0 g_{\text{GUT}}^3$   $b_0 > 0$ 

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- Typical numbers for C's and  $f's^2 \implies \left|G_{4\mathsf{F},*}^{qqql}\right|^2 \lesssim 10^{-7}$

<sup>2</sup>Eichhorn/Held/Wetterich '18

# Discussion IV: *B*-symmetry in ASQG

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- Corollary (strict AS limit):  $k_{\sf UV} o \infty \implies G_{\sf 4F}^{qqql}(M_{\sf QG}) o 0$
- In FP language:  $G^{qqql}_{4F,*} = 0$  and  $G^{qqql}_{4F} \neq 0$  is irrelevant perturbation

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•  $G_{4F,*}^{qqql} = 0 \Rightarrow ASQG = B$ -conserving UV completion of GR

ightarrow Truncation-independent 'proof': Use Quantum Action Principle for regularized effective action  $\Gamma_k$ 

$$e^{-\Gamma_{k}[\Phi]} = \int \mathcal{D}\tilde{\Phi} e^{-S[\Phi] + (\tilde{\Phi}_{X} - \Phi_{X})\Gamma_{k,}^{X}[\Phi] - \frac{1}{2}\mathcal{R}_{k}^{XY}(\tilde{\Phi}_{X} - \Phi_{X})(\tilde{\Phi}_{Y} - \Phi_{Y})}$$
  
$$\delta_{\epsilon}\Gamma_{k}[\Phi] = \left\langle \delta_{\epsilon} \left( S[\tilde{\Phi}] - (\tilde{\Phi}_{X} - \Phi_{X})\Gamma_{k,}^{X}[\Phi] + \frac{1}{2}\mathcal{R}_{k}^{XY}(\tilde{\Phi}_{X} - \Phi_{X})(\tilde{\Phi}_{Y} - \Phi_{Y}) \right) \right\rangle_{k;\Phi}$$

with

$$\left\langle \mathcal{F}[\tilde{\Phi}] \right\rangle_{k;\Phi} \coloneqq e^{\Gamma_{k}[\Phi]} \int \mathcal{D}\tilde{\Phi} e^{-\mathcal{S}[\Phi] + (\tilde{\Phi}_{X} - \Phi_{X})\Gamma_{k,X}^{-1}[\Phi] - \frac{1}{2}\mathcal{R}_{k}^{XY}(\tilde{\Phi}_{X} - \Phi_{X})(\tilde{\Phi}_{Y} - \Phi_{Y})} \mathcal{F}[\tilde{\Phi}]$$

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N.B.: Assuming reg. preserves B-symmetry, manifest for Dirac fermions (more tricky for Weyl!)

$$\delta_{\epsilon}S = 0 \implies \delta_{\epsilon}\Gamma_k = 0$$

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e.g., what do ASQG black holes 'look like'? How about their dynamics? → difficult from first principles – see however Pawlowski/Tränkle '24; usually based on 'RG improvement' of classical solutions Bonanno/Reuter '99, '00, '06; Reuter/Weyer '04; Cai/Easson '10; Liu *et al.* '12; Falls *et al.* '12; Falls/Litim '14; Koch/Saueressig '13, '14; Saueressig *et al.* '15; González/Koch '16; Torres '17; Adéìféoba *et al.* '18; Held *et al.* '19; Bosma *et al.* '19; Platania '20; Bonanno *et al.* '21; Ishibashi *et al.* '21; Borissova/Platania '23; ...

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More generally: Horizons 'eat' global charge

 $\longrightarrow$  Now: Can these 'problematic' contribs to gravitational path integral be suppressed by non-minimal curvature terms?

Borissova/Eichhorn/S.R. Class. Quant. Grav. '25

Assume gravitational path integral given by  $\int Dg e^{iS_{eff}(g)}$ .

Can (quasi-)local contribution to Lagrangian  $\int_x \mathcal{L}_{hor}(g(x))\mu_g(x) \subset S_{eff}(g)$   $(\mu_g(x) = \sqrt{-\det(g(x))} d^4x)$  be found so that  $S_{eff}(g) \to \infty$  if g has horizon? Similar to: Higher-order curvature terms in  $S_{eff} \Rightarrow S_{eff}$  divergent for (many) types of g with curvature singularities cf. Borissova/Eichhorn '21, Borissova '24

#### Claim:

$$\mathcal{L}_{\mathrm{hor}} = \frac{(C^2)^8}{[4C^2(\nabla C)^2 - (\nabla C^2)^2]^2} \qquad C - \text{Weyl tensor w.r.t.}\, g$$

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3. Compute explicitly for  $ds^2 = -(1 + cr^2)dt^2 + a(t)^2(dr^2 + r^2d\Omega^2)$ Finite for all *c*, vanishes for Weyl-flat  $c \to 0$  (flat Minkowski  $g = \eta a$  fortiori)

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• Usual disclaimers (unitarity, stability, ...) apply

# Summary and outlook

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 Horizons (i.e. those that 'eat' global charge) may potentially be suppressed in path integrals using actions that are only mildly non-local Unified construction for general horizons? Wormholes/topology change? Black-hole entropy? Derivation from more fundamental (e.g., discrete) approach or integrating out DOFs?

## Acknowledgement

#### Based on:

*Phys. Lett. B* **850**, 138529 (2024) (w/ A. Eichhorn) *Class. Quantum Grav.* **42**, 037001 (2025) (w/ J. Borissova & A. Eichhorn)

#### Collaborators



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