Towards Scattering Amplitudes in Lorentzian Asymptotically Safe Quantum Gravity

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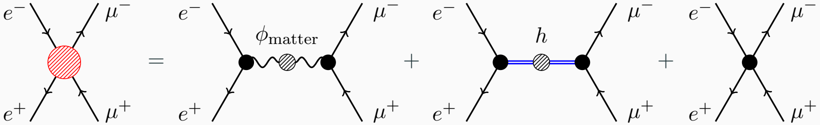
Tree-level graviton-mediated scattering



[Pastor-Gutiérrez, Pawlowski, MR, Ruisi '24]

- Tree-level cross section

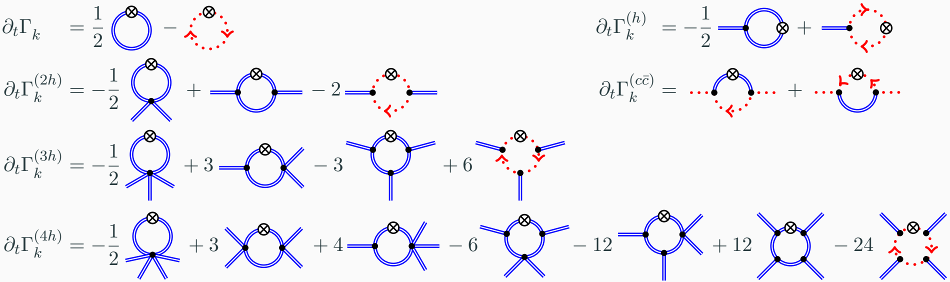
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha_e^2}{4s}(1+\cos^2\theta) + \frac{G_\mathrm{N}\,\alpha_e}{4}\cos^3\theta + \frac{G_\mathrm{N}^2}{64}s(1-3\cos^2\theta+4\cos^4\theta)$$

- Violates unitarity bounds $\sigma \propto s\,G_\mathrm{N}^2$

- Naive RG improvement

$$\sigma \propto s\,G_\mathrm{N}(s)^2 = s\frac{1}{s}^2 = \frac{1}{s}$$

Fluctuation approach – expansion in scattering vertices



[Pure gravity: Christiansen, Knorr, Pawlowski, Rodigast '14; Christiansen, Knorr, Meibohm, Pawlowski, MR '15; Denz, Pawlowski, MR '16; Christiansen, Falls, Pawlowski, MR '17; Knorr, Schiffer '21; ...]
[Gravity-matter: Meibohm, Pawlowski, MR '15; Christiansen, Litim, Pawlowski, MR '17; Eichhorn, Lippoldt, Schiffer '18; ...]
[Reviews: Pawlowski, MR '20; '23]

- Flat background $g_{\mu\nu} = \delta_{\mu\nu} + \sqrt{G_\mathrm{N}}\, h_{\mu\nu}$ or $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_\mathrm{N}}\, h_{\mu\nu}$
- Infinite tower of coupled integral-differential equations
- Convenient approach to study asymptotic safety without derivative expansion

[See also: Form factor approach Knorr, Ripken, Saueressig '22; ...]

A short guide to fluctuation computations

Momentum-dependent coupling functions inspired by EH action $S_{\mathrm{EH}} \propto \frac{1}{G_{\mathrm{N}}} \int_x (2\Lambda - R)$

Graviton two-point function

$$\Gamma^{(hh)}_{k,\mu\nu\rho\sigma}(p) \propto Z_{h,\mathrm{tt}}(p)(p^2 - 2\Lambda_2)\mathcal{T}_{\mathrm{tt},\mu\nu\rho\sigma}(p) + 4 \text{ more tensor structures}$$

Graviton three-point function at momentum symmetric point $|p_1| = |p_2| = |p_3| = p$

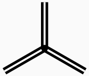
$$\Gamma^{(hhh)}_{k,\mu\nu\rho\sigma\kappa\lambda}(p) \propto Z^{3/2}_{h,\mathrm{tt}}(p) \left(\sqrt{G_{\mathrm{N},3}(p)}\, p^2 + \#\Lambda_3 \right) \mathcal{T}_{\mathrm{tt},\mu\nu\rho\sigma\kappa\lambda}(p) + 32 \text{ more tensor structures}$$

Solve tower of coupled integral differential equation

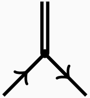
$$\partial_t \Gamma^{(hh)}(p) \propto \int \mathrm{d}^4 q\, F[\Gamma^{(2)}(p,q), \Gamma^{(3)}(p,q), \Gamma^{(4)}(p,q)]$$

Typical approximations: TT tensor structure, $G_{\mathrm{N},n>n_{\mathrm{max}}}(p) = G_{\mathrm{N},3}(p)$, $\Lambda_{n>n_{\mathrm{max}}} = \Lambda_3$, ...

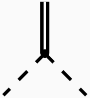
Avatars of couplings



$\longrightarrow G_3(p_1, p_2, p_3)$



$\longrightarrow G_\psi(p_1, p_2, p_3)$



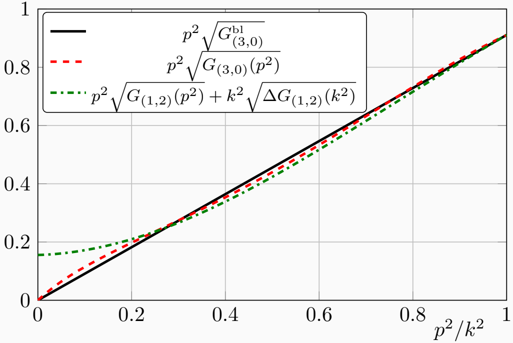
$\longrightarrow G_\varphi(p_1, p_2, p_3)$

- Related by symmetry identities
- Reduce to G_N + higher-order terms for $k \to 0$

Effective universality: $G_3 \approx G_\psi \approx G_\varphi \approx \dots$ at FP

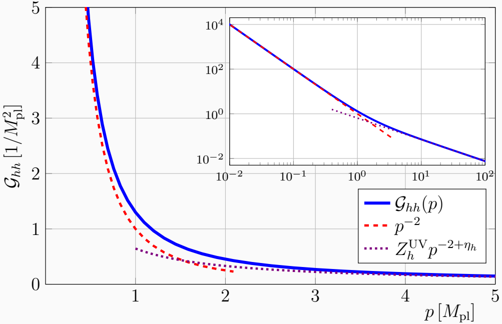
[Eichhorn, Labus, Pawlowski, MR '18]

[Eichhorn, Lippoldt, Pawlowski, MR, Schiffer '18]

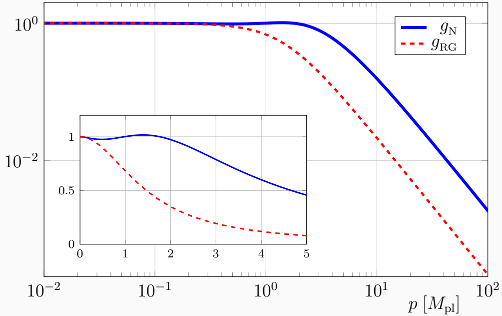


Graviton correlation functions at $k = 0$

Propagator



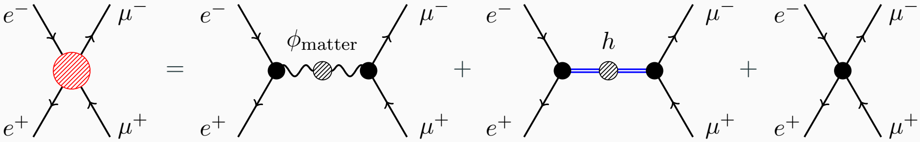
Three-graviton vertex



[Bonanno, Denz, Pawlowski, MR '21]

- Momentum dependent correlation functions integrated to $k = 0$
- RG scale and momentum dependence agree qualitatively

Full graviton-mediated scattering



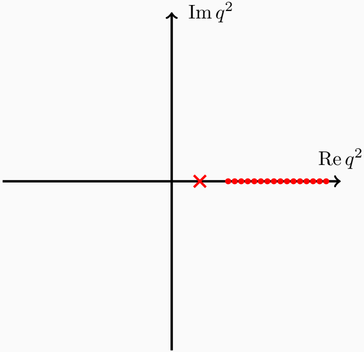
Need:

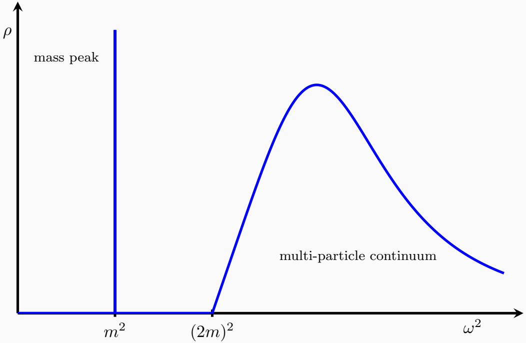
- well-behaved propagators without ghost or tachyonic instabilities
- access to correlation functions on Lorentzian signature at time-like momenta

Approach: direct Lorentzian & analytic continuation from Euclidean

Källén-Lehmann spectral representation [Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$

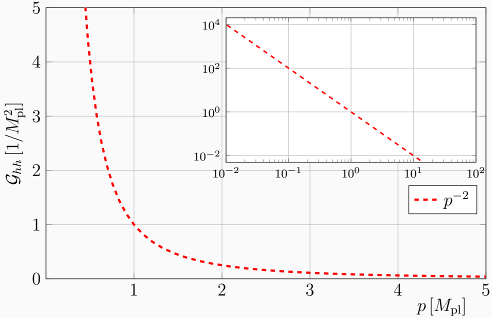


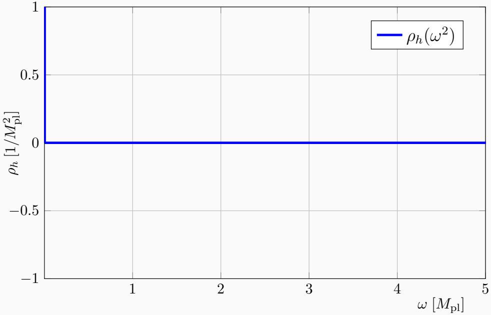


Classical graviton spectral function

Einstein-Hilbert action: $S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$

Flat Minkowski background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



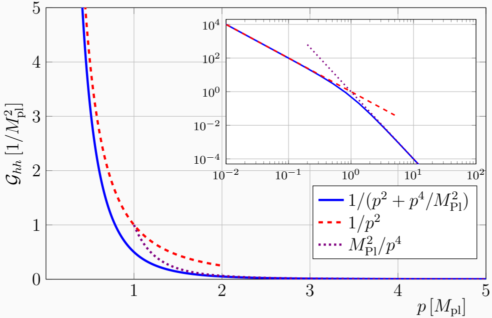


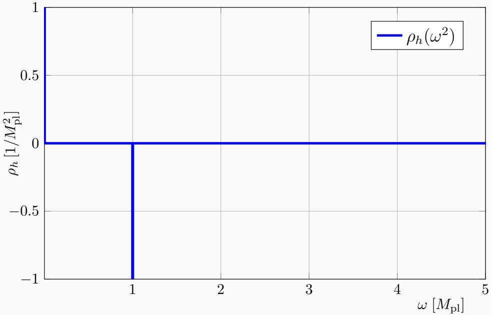
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

Higher-derivative action: $S_{\mathrm{HD}} = S_{\mathrm{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$





$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\mathrm{Pl}}^2 + p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\mathrm{Pl}}^2)$$

- Callan-Symanzik cutoff preserves causality and Lorentz invariance

$$R_k = Z_\phi \, k^2$$

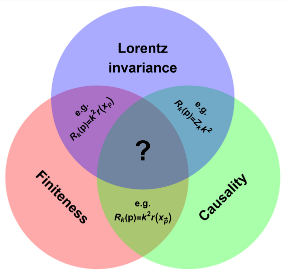
- Finite flow equation with counterterms

$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k \, \partial_t R_k - \partial_t S_{\mathrm{ct},k}}$$

- Dim reg of UV divergences in $d = 4 - \varepsilon$ possible
- Use spectral function in flow diagrams

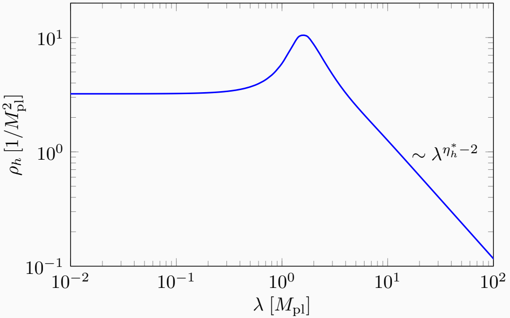
$\partial_t \rho_h \propto$  $+ \ldots$ with $\displaystyle \mathcal{G}_h(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$

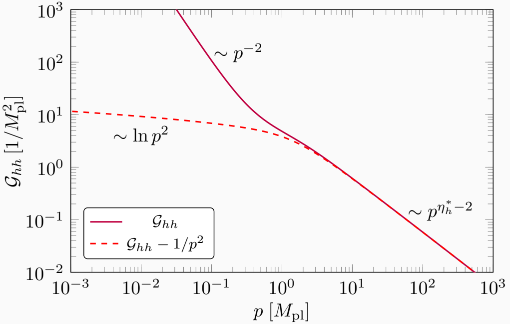
$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$$



[fQCD, MR ’22]

Graviton spectral function

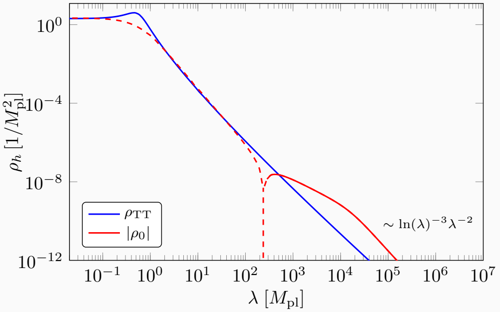


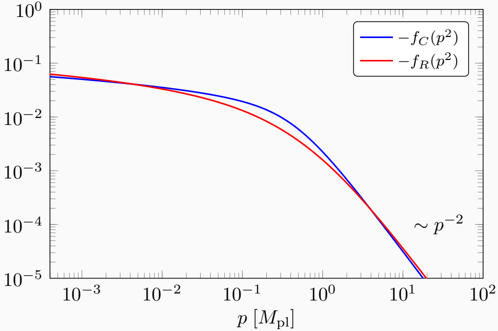


[Fehre, Litim, Pawlowski, MR '21]

- Massless graviton delta-peak with positive multi-graviton continuum
- No ghosts and no tachyons $\longrightarrow$ no indications for unitarity violation
- Good agreement with reconstruction results and EFT [Bonanno, Denz, Pawlowski, MR '21]

Graviton spectral function – TT and scalar graviton mode

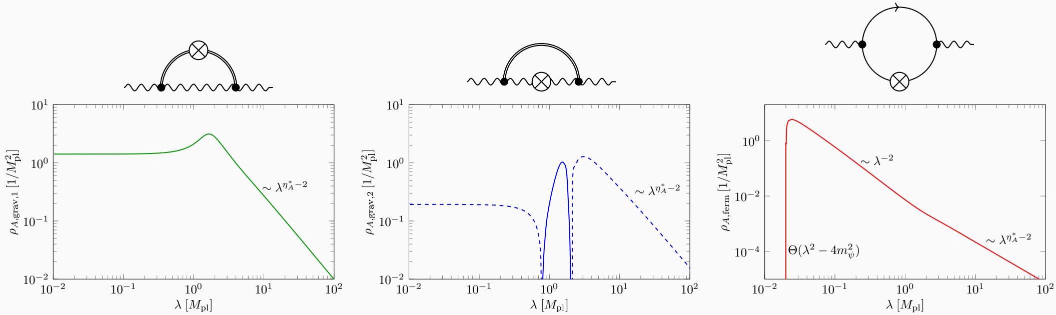




[Assant, Litim, MR *(in prep)*]

See poster and talk(!) by Gabriel Assant

Photon and scalar spectral function with gravity fluctuations



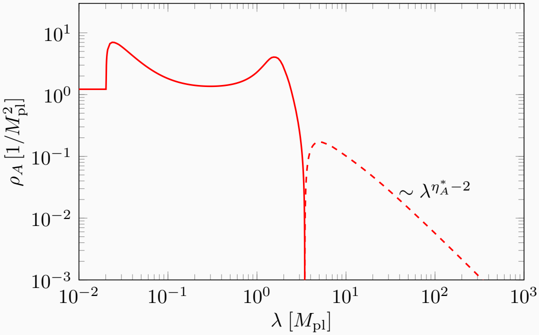
[King, Kher, MR *(in prep)*]

- Ansatz for spectral function

$$\rho_A = \frac{1}{Z_A}\Big[2\pi\delta(\lambda^2 - m_A^2) + \theta(\lambda^2 - (m_A + m_h)^2) f_{A,\mathrm{grav}}(\lambda) + \theta(\lambda^2 - 4m_\psi^2) f_{A,\mathrm{ferm}}(\lambda)\Big]$$

- Photon and scalar structurally similar
- Only diagram with matter regulator leads to negative contributions

Photon and scalar spectral function with gravity fluctuations



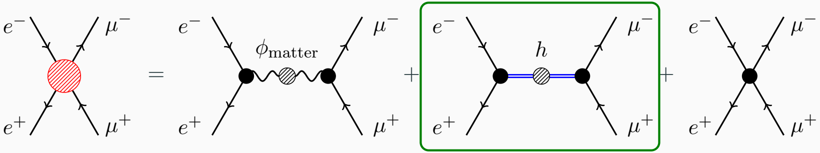
[King, Kher, MR *(in prep)*]

- Positive spectral function for $\eta_h < 0.3$ or with enough fermions
- Gravity contribution is gauge dependent

$$\rho_{A,\text{IR, gravity}} = -\frac{8\left(\beta^2 - 3\alpha\right)}{3(\beta-3)^2}$$

- Direct access to form factors $f_A(p^2)$ and $f_\varphi(p^2)$

Non-perturbative graviton-mediated scattering



- Replace $S^{(n)} \longrightarrow \Gamma^{(n)}$
- Full matrix element of graviton-mediated diagram

$$\mathcal{M}_h \propto s\, G^{\frac{1}{2}}_{\mathrm{N},h\bar{\psi}\psi}(p_{e^+}, p_{e^-}, p_h)\, G^{\frac{1}{2}}_{\mathrm{N},h\bar{\psi}\psi}(p_{\mu^+}, p_{\mu^-}, p_h)$$

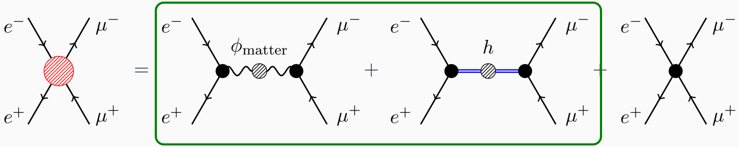
- Approximation: momentum-symmetric point & effective universality

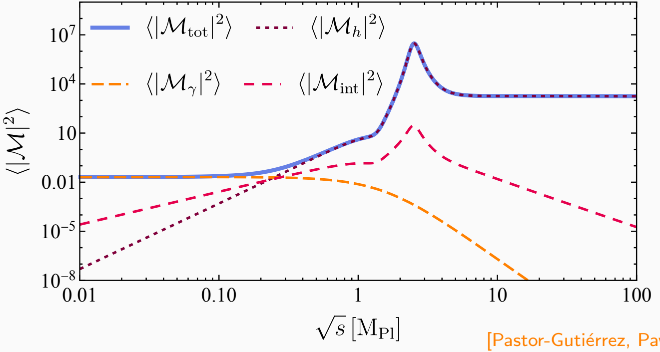
$$G_{\mathrm{N},h\bar{\psi}\psi}(\mathbf{p}) \longrightarrow G_{\mathrm{N},3h}(p^2)$$

- Analytic continuation from Euclidean data [Bonanno, Denz, Pawlowski, MR '21]

$$G_{\mathrm{N},3h}(p_E^2) \xrightarrow[\text{continuation}]{\text{analytic}} G_{\mathrm{N},3h}(p^2)$$

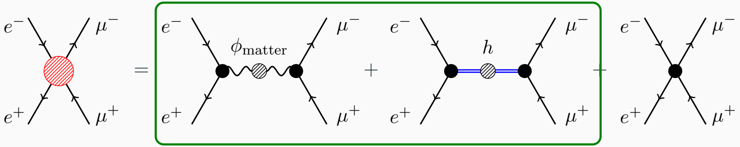
Towards graviton-mediated scattering cross-sections

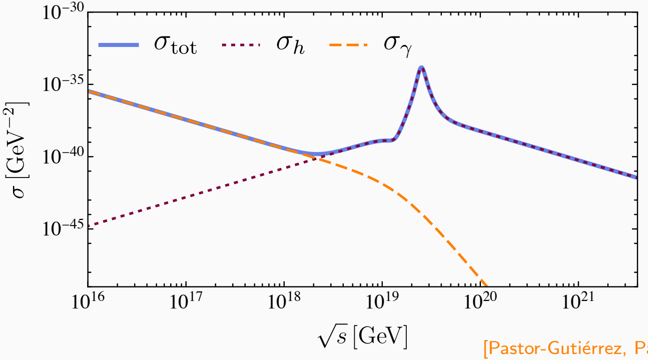




[Pastor-Gutiérrez, Pawlowski, MR, Ruisi '24]

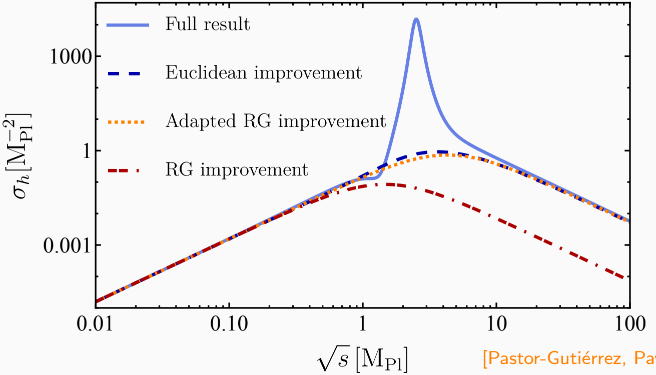
Towards graviton-mediated scattering cross-sections





[Pastor-Gutiérrez, Pawlowski, MR, Ruisi '24]

Comparison with RG improvement

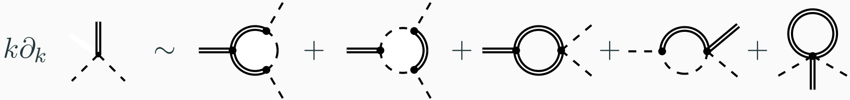


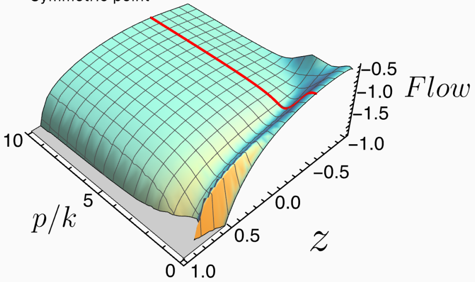
[Pastor-Gutiérrez, Pawlowski, MR, Ruisi '24]

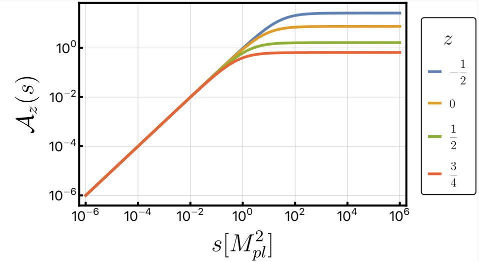
- RG improvement: $G_\mathrm{N}(k,0) \longrightarrow G_\mathrm{N}(\sqrt{s},0)$
- Euclidean improvement: $G_\mathrm{N}(0,p^2) \longrightarrow G_\mathrm{N}(0,s)$
- Adapted RG improvement: $G_\mathrm{N}(s) = \frac{g^*}{s+g^*\,\mathrm{M}_{\mathrm{Pl}}^2}$ with $g^* = G_\mathrm{N}(0,p^2 \to \infty)$

Outlook: Momentum dependent vertex flows

Momentum dependent vertex flow with $p = p_{\varphi,1} = p_{\varphi,2}$ and general angle $z = \frac{\vec{p}_{\varphi,1}\cdot\vec{p}_{\varphi,2}}{p_{\varphi,1}p_{\varphi,2}}$







[Pawlowski, Portas Chiesa, MR, *(in prep)*]

See poster by Angelo Portas Chiesa

Summary

- Asymptotic safety in Lorentzian signature
- Well-behaved spectral functions without ghost or tachyonic instabilities
- Cross-section from analytic continuation of Newton coupling compatible with unitarity
- Fully momentum dependent vertex couplings on the way

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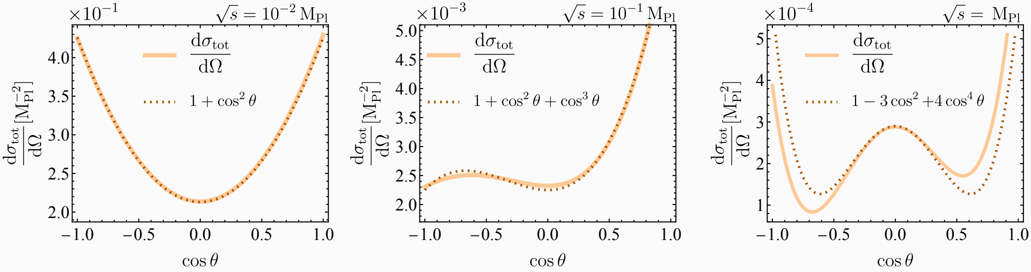
Thank you for your attention!

Back-up slides

Angular dependence



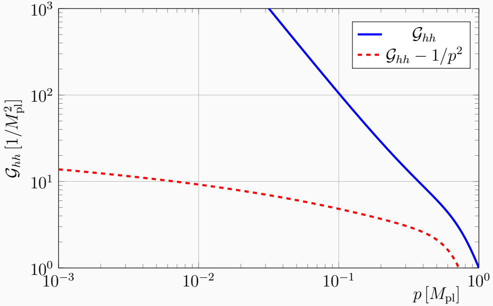
[Pastor-Gutiérrez, Pawlowski, MR, Ruisi '24]

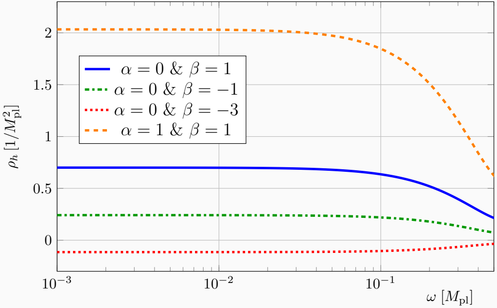
Angular dependence in accordance with expectation from spin of mediator

EFT graviton spectral function

One-loop effective action: $\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\left(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}\right) + \ldots$

Gauge-fixing $\quad S_{\text{gf}} = \frac{1}{\alpha}\int_x F_\mu^2 \quad$ with $\quad F_\mu = \bar\nabla^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar\nabla_\mu h^\nu{}_\nu$





[Pawlowski, MR '23]

$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)p^4}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2) + \text{const.} + \ldots$$