Non-vanishing Yukawa interactions in asymptotically safe quantum gravity

Quantum Spacetime and the Renormalization Group

Heidelberg, April 1, 2025

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In collaboration with

G. de Brito, M. Reichert: **arXiv: 2504.XXXXX**

Radboud Universiteit

Towards the Yukawa sector of the Standard Model

Overall goal: test if Landau poles of SM can be cured with ASQG

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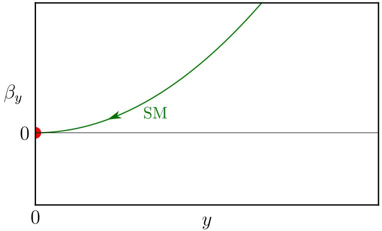

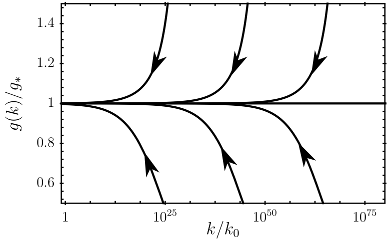

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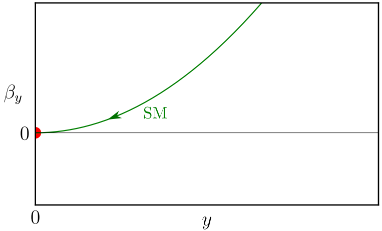

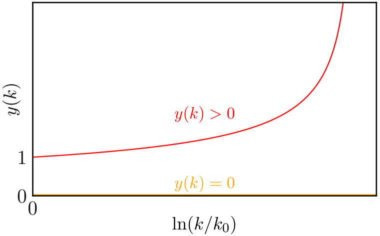

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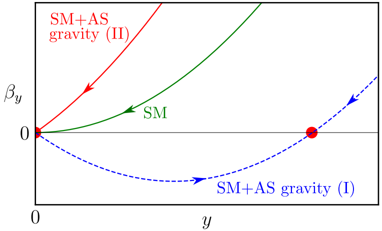
$$g_{\mathrm{N}}\, f_y \begin{cases} = \text{const.} > 0\,, & \text{for } k > M_{\mathrm{Pl}} \wedge \Lambda < \Lambda_{\mathrm{crit}} \\ = \text{const.} < 0\,, & \text{for } k > M_{\mathrm{Pl}} \wedge \text{for } \Lambda > \Lambda_{\mathrm{crit}} \\ \approx 0\,, & \text{for } k < M_{\mathrm{Pl}} \end{cases}$$



[Oda, Yamada; 2015], [Eichhorn, Held, Pawlowski; 2016], [Eichhorn, Held; 2017]

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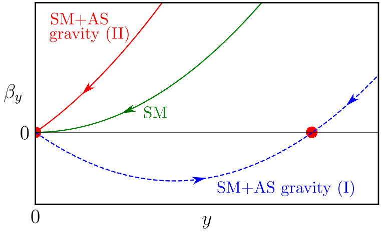
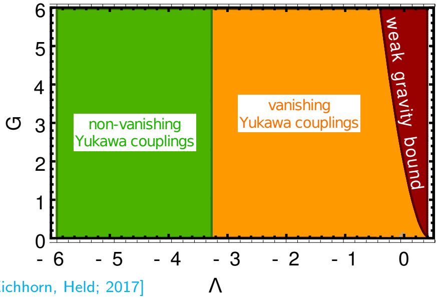


[Oda, Yamada; 2015], [Eichhorn, Held, Pawlowski; 2016], [Eichhorn, Held; 2017]

UV completion of the simple Yukawa system: constraints on gravitational dynamics

Additionally: top mass might be retro-dicted [Eichhorn, Held, 2017]

Simple Yukawa system: state of the art (LO)



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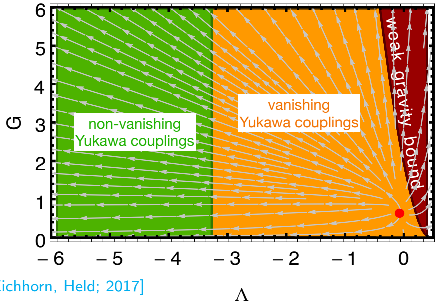

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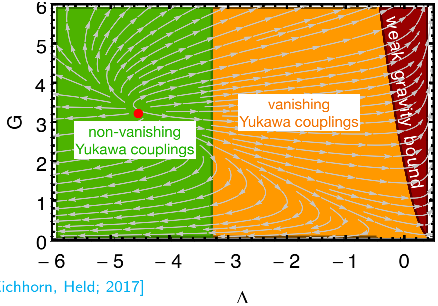

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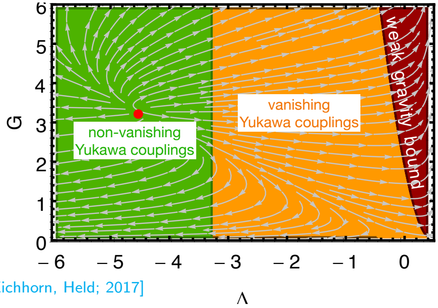

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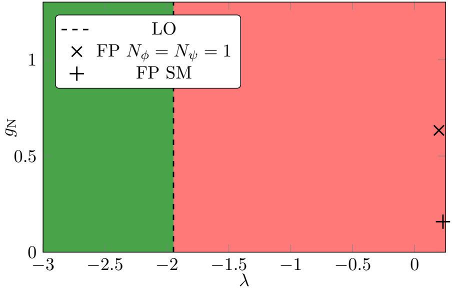

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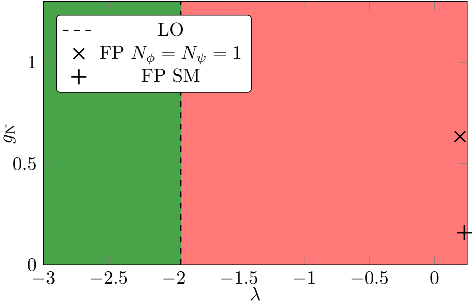
  $+$ retrodiction [Eichhorn, Held; 2017]
- But: SM fixed point: large anomalous dimensions
- Different scheme: (fluctuation computations) $f_{y,*} < 0$
  see, e.g., [Christiansen et al.; 2019]

Possible ways out?



1. FP shifts when including higher-order operators
   see e.g. [Eichhorn, MS; 2022], [Pawlowski, Reichert; 2023] for reviews
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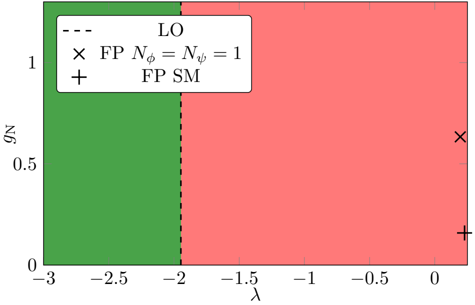

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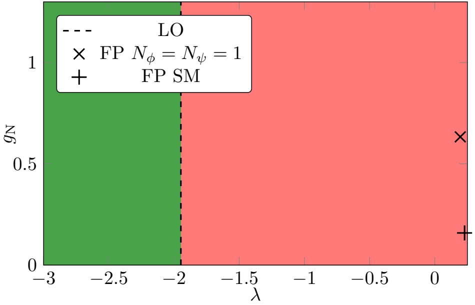

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2. Region of relevance shifts when including higher-order operators
   - more precisely, need $\Theta_y > 0$, for $y_{\mathrm{IR}} > 0$

$$\Theta_I = -\mathrm{eig}\left(M_{ij}\right)\,, \quad \text{with} \quad M_{ij} = \left(\frac{\partial \beta_{g_j}}{\partial g_i}\right)\Bigg|_{\mathbf{g}=\mathbf{g}^*}$$

Contributions beyond LO

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- At LO: $\Theta_y = f_y g_{\mathrm{N}}$;
- NLO: g_{N}^2 contributions to Θ_y;
- Possible sources:
  - direct contributions to M_{11}
    - contribution $\sim y\, g_{\mathrm{ind}}$ to β_y with $g_{\mathrm{ind},*} \neq 0$
  - off-diagonal contributions in M_{ij} - contribution to β_y can be independent of y itself

Direct contributions: induced operators

$$\beta_y = \Big(-f_y \, g_N \qquad\qquad \Big) y + \#_{SM} \, y^3 + \mathcal{O}(y^4)$$

- Example: Chiral fermion ψ

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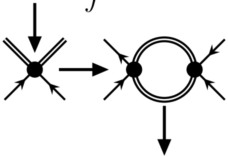
  - From kinetic term:

$$\Gamma_k^{\mathrm{kin}} = \int \mathrm{d}^4x \sqrt{g}\ \left(\bar{\psi} \not{D} \psi\right)$$



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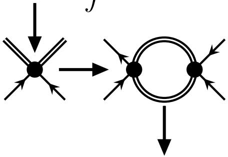

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- Schematically:

$$\beta_{\lambda_1} = C_0(g_{\mathrm{N}}) + C_1(g_{\mathrm{N}})\, \lambda_1 + C_2\, \lambda_1^2$$

- Interacting fixed point: $\lambda_{1,*} \sim g_{\mathrm{N}}^2$

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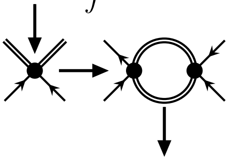

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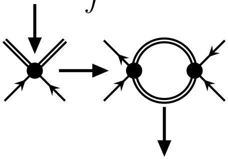

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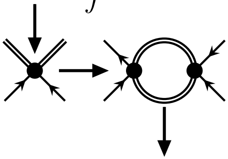

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- In general: two kinds of such contributions $g_{\mathrm{self},*} \sim g_{\mathrm{N}}^2$, $g_{\mathrm{n-m},*} \sim g_{\mathrm{N}}$
- g_{ind} does not induce y $\Rightarrow$ $\beta_{g_{\mathrm{ind}}}$ do not contribute to Θ_y via M_{ij}

Off-diagonal contributions

- Consider new coupling g_{ho} such that

$$M = \begin{bmatrix} -f_y\, g_{\mathsf{N}} + \#_{\mathrm{ind}}\, g_{\mathsf{N}}^2 & \#_1\, g_{\mathsf{N}} \\ \#_2\, g_{\mathsf{N}} & \dim_{\mathrm{ho}} + \#_3\, g_{\mathsf{N}} \end{bmatrix}.$$

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- This yields

$$\Theta_y = f_y\, g_{\mathsf{N}} + g_{\mathsf{N}}^2 \left(\frac{\#_1\, \#_2}{\dim_{\text{ho}}} - \#_{\text{ind}} \right) + \mathcal{O}(g_{\mathsf{N}}^3)$$

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- Includes operators with $g_{\rm ho,*} = 0$!

Simple Yukawa system: Summary of NLO Operators

- **contributions to $\mathbf{M}_{\mathbf{11}}$**
- contributions to β_y with $g_* \neq 0$

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Vertex	Operator	Coupl.	Dim.
$h_{\mu\nu}\bar{\psi}\psi$	$R^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi$	σ_{Ric}	6
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	$R\,\partial_\mu\phi\partial^\mu\phi$	ρ_{R}	6
$(\bar{\psi}\psi)^2$	$(\bar{\psi}\gamma_\mu\psi)^2$	λ_{V}	6
	$(\bar{\psi}\gamma_\mu\gamma_5\psi)^2$	λ_{A}	6
$\phi^2\bar{\psi}\psi$	$\partial_\mu\phi\partial^\mu\phi\,\bar{\psi}\not{D}\psi$	$\chi_{1/2}$	8
ϕ^4	$(\partial_\mu\phi\partial^\mu\phi)^2$	K_2	8

see [Eichhorn, MS; 2022] for references

contribute via η_i only $\Rightarrow$ neglect

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	$C^2_{\mu\nu\rho\sigma}\,\phi\bar{\psi}\psi$	y_{C^2}	8

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Induced operators shift FP, but admit lower-triangular stability sub-matrix

Simple Yukawa system: Summary of NLO Operators

- **contributions to $\mathbf{M_{11}}$**
- contributions to β_y with $g_* \neq 0$

Vertex	Operator	Coupl.	Dim.
$h_{\mu\nu}\bar{\psi}\psi$	$R^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi$	σ_{Ric}	6
	$R\,\bar{\psi}\not{D}\psi$	σ_{R}	6
$h_{\mu\nu}\phi^2$	$R^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi$	ρ_{Ric}	6
	$R\,\partial_\mu\phi\partial^\mu\phi$	ρ_{R}	6
$(\bar{\psi}\psi)^2$	$(\bar{\psi}\gamma_\mu\psi)^2$	λ_{V}	6
	$(\bar{\psi}\gamma_\mu\gamma_5\psi)^2$	λ_{A}	6
$\phi^2\bar{\psi}\psi$	$\partial_\mu\phi\partial^\mu\phi\,\bar{\psi}\not{D}\psi$	$\chi_{1/2}$	8
ϕ^4	$(\partial_\mu\phi\partial^\mu\phi)^2$	K_2	8

see [Eichhorn, MS; 2022] for references

contribute via η_i only $\Rightarrow$ neglect

- **contributions to $\mathbf{M_{ij}}$**
- contributions to β_y with $\hat{g}_{i,*} = 0$
- if $\beta_y \supset \hat{g}_i$ and $\beta_{\hat{g}_i} \supset y$: non-vanishing contribution to Θ_y

Vertex	Operator	Coupl	Dim.
$\phi\bar{\psi}\psi$	$\Box\phi\bar{\psi}\psi$	$y_{\Box,1}$	6
	$\phi\bar{\psi}\Box\psi$	$y_{\Box,2}$	6
$h_{\mu\nu}\phi\bar{\psi}\psi$	$R\,\phi\bar{\psi}\psi$	y_{R}	6
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Induced operators shift FP, but admit lower-triangular stability sub-matrix

Modulo momentum-dependences: No further g_{N}^2 contribution to Θ_y

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$$\theta_i = (1.43 \pm 5.26i, -3.15, -3.19, -4.17, -4.61 \pm 0.32i, -4.86)\,.$$

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- both cases: well-behaved extensions of the Reuter FP!

Critical exponent of the Yukawa

- Plug FP-values into non-induced operators, and compute critical exponents at

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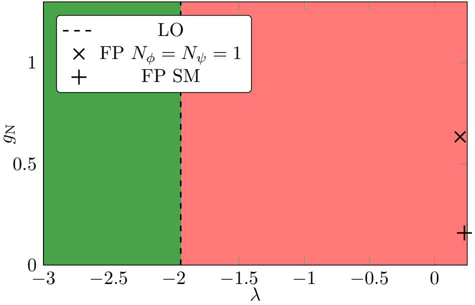
UV completion of the simple Yukawa system via NLO operators!

Yukawa coupling at NLO



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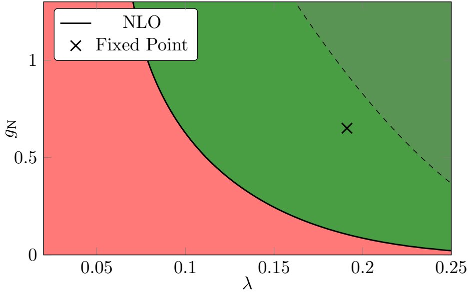

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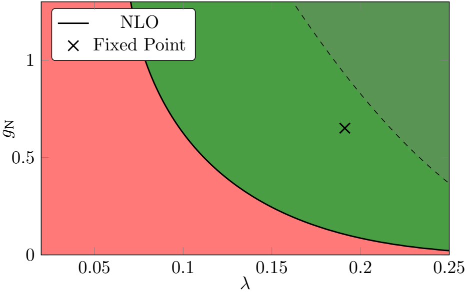

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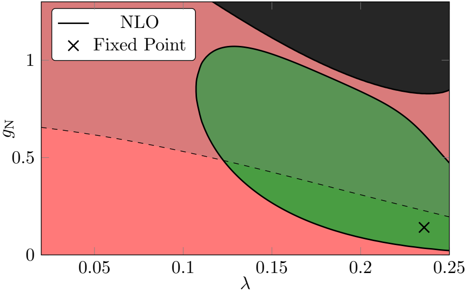

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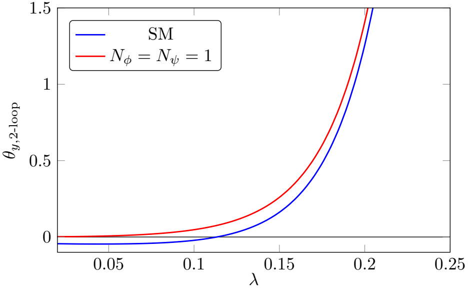

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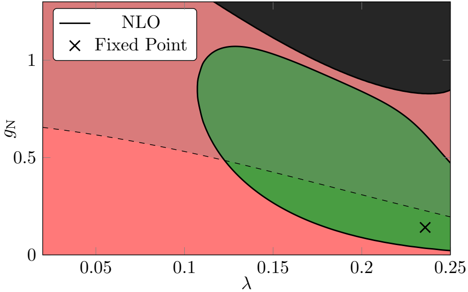

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- Fixed point lies inside that new regime!

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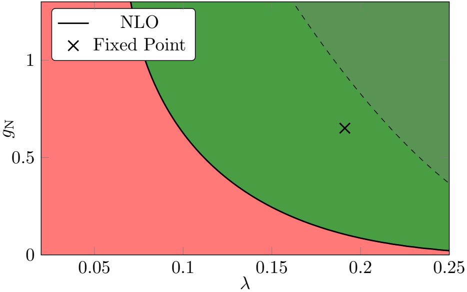

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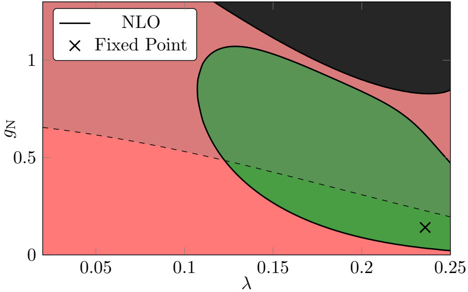

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- LO: $\Theta_y < 0$, NLO: $\Theta_y > 0$; What about NNLO (i.e., g_N^3-contributions)?

Estimating NNLO effects ('error bars')

- previously observed: $R\,\phi\,\bar{\psi}\psi$, $R^2\,\phi\,\bar{\psi}\psi$ and $C^2\,\phi\,\bar{\psi}\psi$ dominate
- estimate effect of $R^3\,\phi\,\bar{\psi}\psi$-type operator

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$$M_{NLO}\big|_{\lambda_*} = \begin{bmatrix} 1.5\,g_{\rm N} & 0.6\,g_{\rm N} & 1.2\,g_{\rm N} & 1.5\,g_{\rm N} & -2.4\,g_{\rm N} \\ 3.0\,g_{\rm N} & 2-0.62\,g_{\rm N} & 1.2\,g_{\rm N} & 0 & 0 \\ -27\,g_{\rm N} & -0.65\,g_{\rm N} & 2+0.006\,g_{\rm N} & 0.42\,g_{\rm N} & 10\,g_{\rm N} \\ 124\,g_{\rm N} & 33\,g_{\rm N} & 49\,g_{\rm N} & 4+5.8\,g_{\rm N} & -57\,g_{\rm N} \\ 43\,g_{\rm N} & 19\,g_{\rm N} & 18\,g_{\rm N} & 46\,g_{\rm N} & 4-9.5\,g_{\rm N} \end{bmatrix}$$

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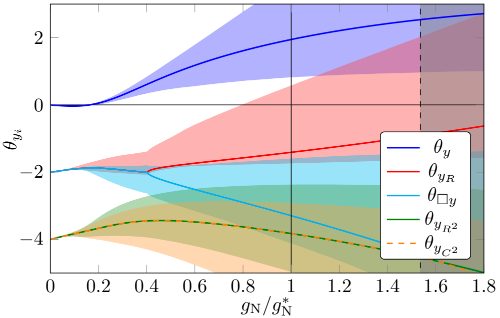
- Simulate effect of NNLO operator on Θ_y (pick appropriate range for $\#_r$)

$$M_{(N)NLO}\big|_{\lambda_*} = \begin{bmatrix} 1.5\,g_\mathrm{N} & -2.1\,g_\mathrm{N} & 3.8\,g_\mathrm{N} & -16\,g_\mathrm{N} & 10.2\,g_\mathrm{N} & 0 \\ -0.86\,g_\mathrm{N} & 2-0.62\,g_\mathrm{N} & -1.1\,g_\mathrm{N} & 0 & 0 & 0 \\ -8.9\,g_\mathrm{N} & 7.2\,g_\mathrm{N} & 2+0.006\,g_\mathrm{N} & 8.9\,g_\mathrm{N} & -21\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} \\ -13\,g_\mathrm{N} & 10.5\,g_\mathrm{N} & -16\,g_\mathrm{N} & 4+4.7\,g_\mathrm{N} & -51\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} \\ -13\,g_\mathrm{N} & 16\,g_\mathrm{N} & -17\,g_\mathrm{N} & 52\,g_\mathrm{N} & 4-8.4\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} \\ \#_\mathrm{r}\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} & \#_\mathrm{r}\,g_\mathrm{N} & 6+\#_\mathrm{r}\,g_\mathrm{N} \end{bmatrix}$$

Results of (N)NLO simulations

- Minimal matter



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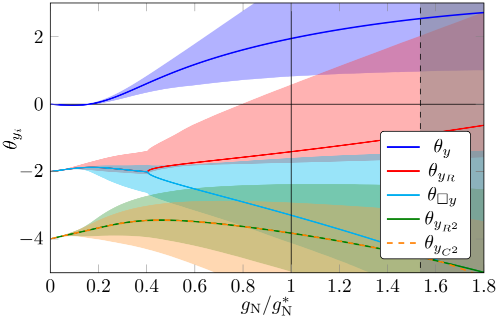

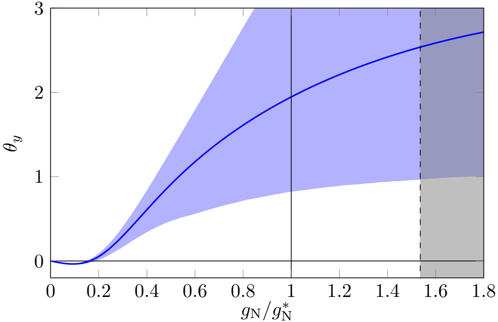

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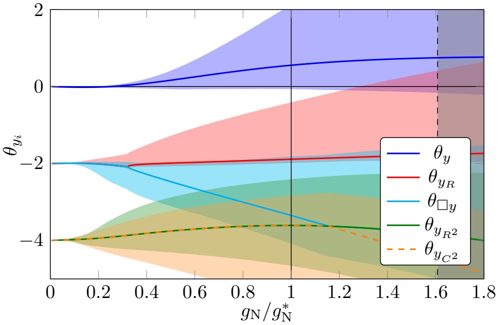

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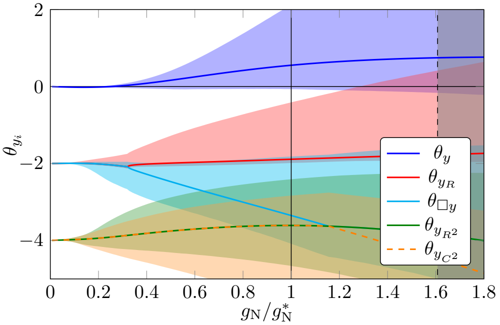

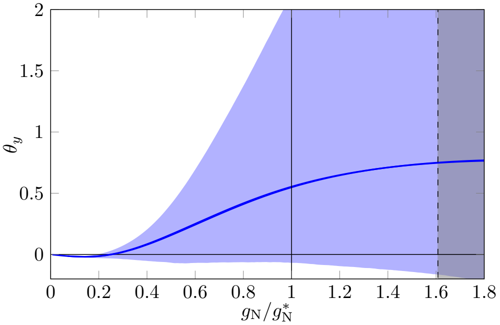

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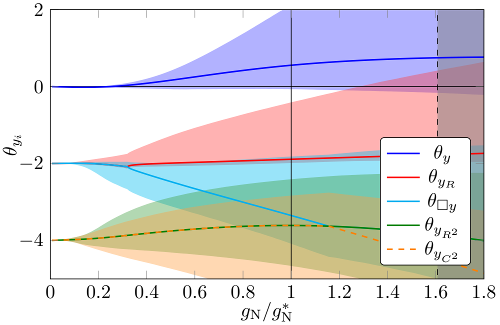

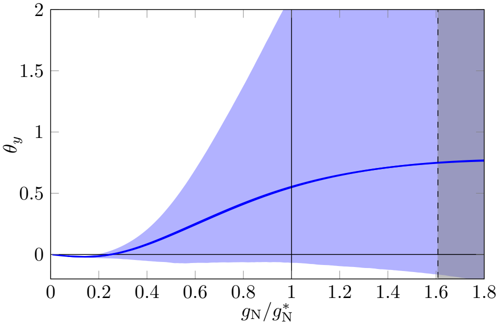

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- (N)NLO simulations: insights into robustness
- but: large(ish) error-bars

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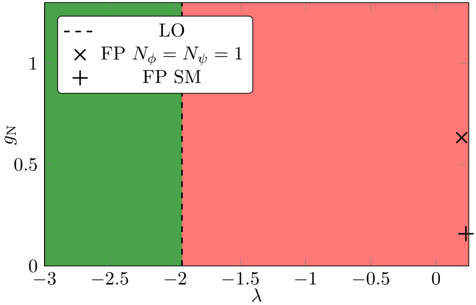

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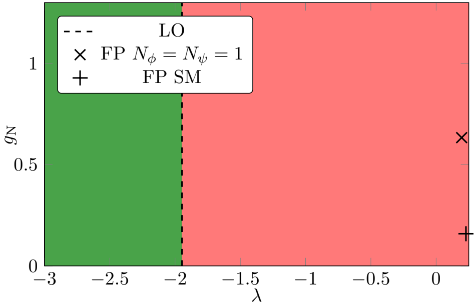

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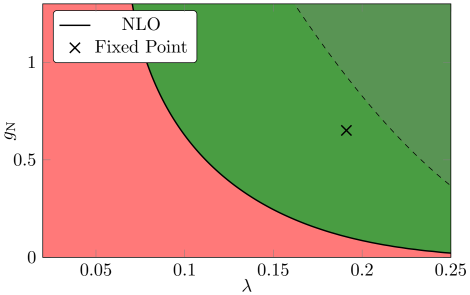

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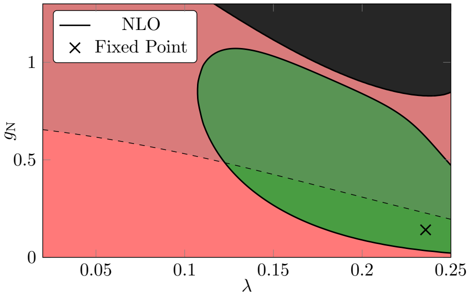

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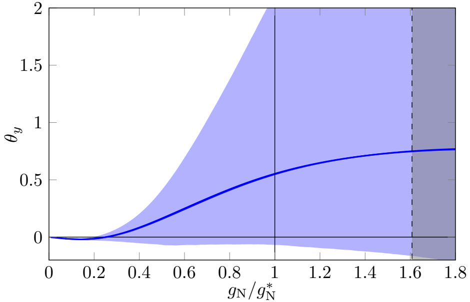

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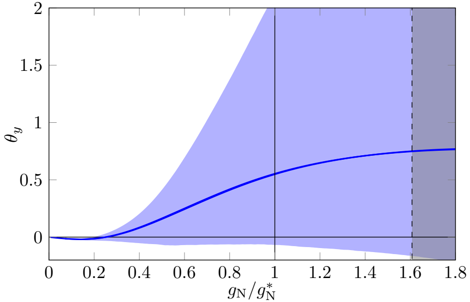

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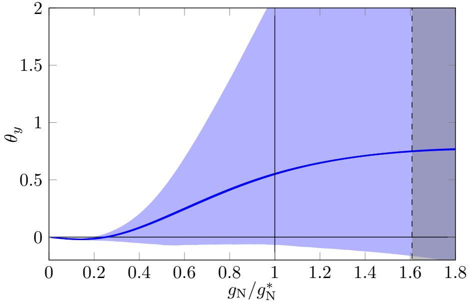

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Thank you for your attention!

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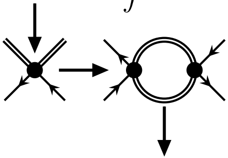

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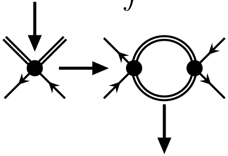

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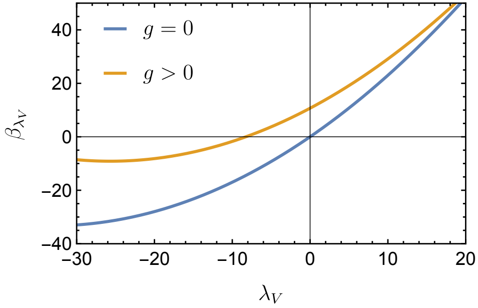

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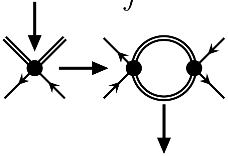

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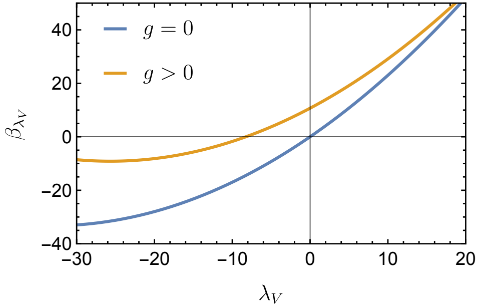

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