Emergence of inflaton potential from asymptotically safe gravity

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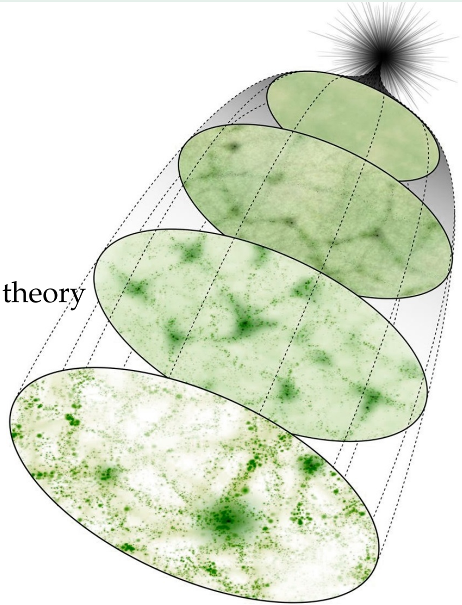
Presentation Overview

1. *(The good, the bad and the ugly)*$^{-1}$
2. Cosmological inflation in a nutshell
3. Renormalization Group Flow of Scalar-Tensor theory
4. Emergence of Inflation from the UV
5. Conclusions



$(\textit{The good, the bad and the ugly})^{-1}$

① **Euclidean Signature**

$\Downarrow$

② **Gauge, regulator and trace method**

$\Downarrow$

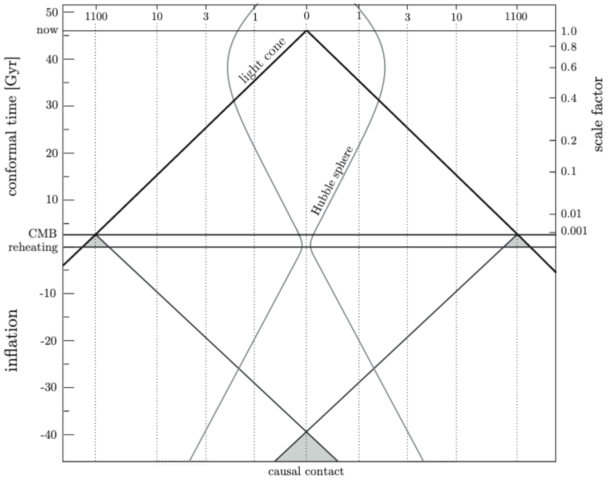
③ **Truncation and approximations**

All calculations in this talk will be performed in euclidean signature, and in the end it will be assumed that observables at $k = 0$ are independent of this.

$\downarrow$

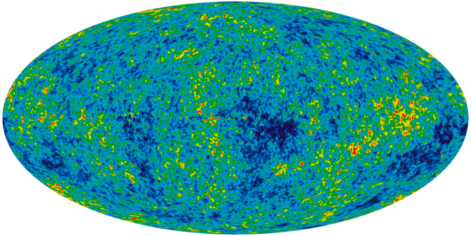
Background computation, exponential splitting of the metric, “physical gauge”, field dependent regulator, heat kernel method, truncations ...

Cosmological Inflation in a nutshell



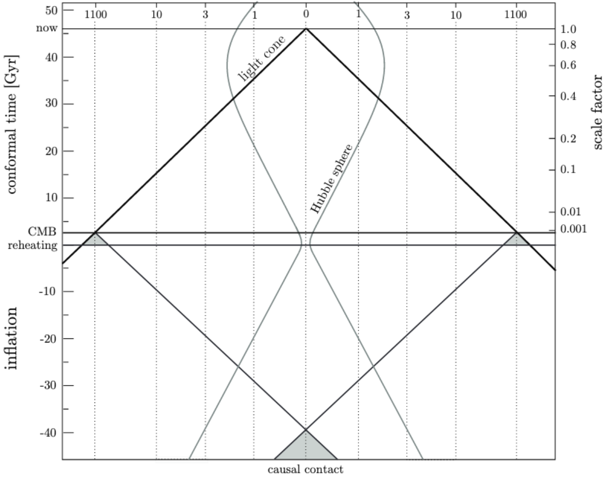
1

Cosmic Microwave Background (CMB)



[1] Credit images: D. Baumann Lecture notes in cosmology https://cmb.wintherscoming.no/pdfs/baumann.pdf

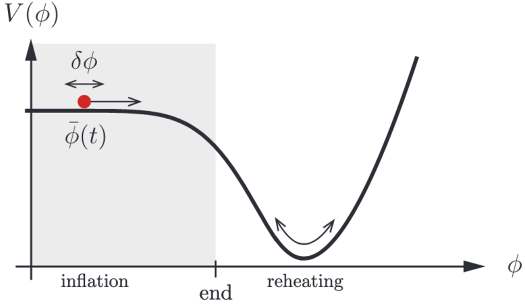
Cosmological Inflation in a nutshell



2

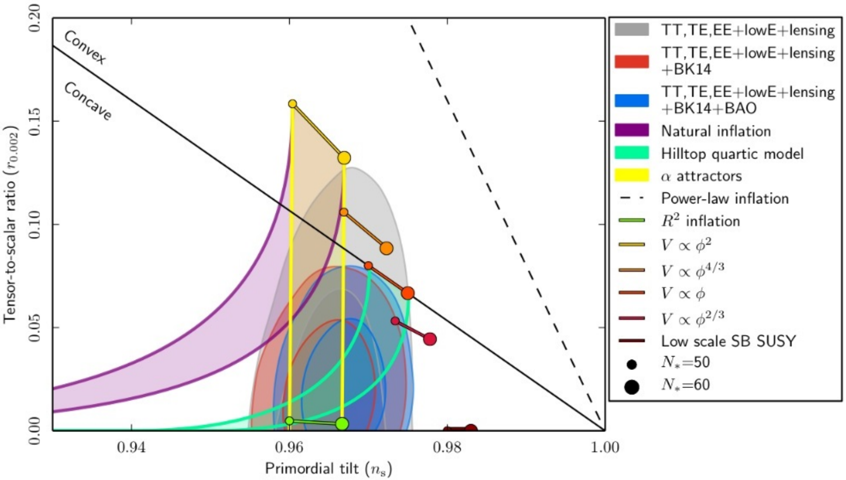
Slow-Roll inflation

$$\Gamma = \int d^4x\sqrt{g}(-\frac{R}{16\pi G} + V(\phi) + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi)$$



[2] Credit images: D. Baumann Lecture notes in cosmology https://cmb.wintherscoming.no/pdfs/baumann.pdf

Cosmological Inflation - Some open questions



[3]

- Where do these models come from? Can we build one from fundamental physics?
- Can we predict the initial conditions of the inflaton field?

$\Downarrow$

Asymptotic Safety (?)

[3] Credit image: Planck 2018 results. X. Constraints on inflation. arXiv:1807.06211

How is this story connected to Asymptotic Safety?

As you probably guessed, the story is connected by a RG flow

$$\Gamma(k=\infty) = \textit{UV fixed point}$$
$$\Downarrow$$
$$\Gamma(k=0) = \textit{Full effective action}$$

$\rightarrow$ Study the phenomenology of the emergent $\Gamma(k=0)$

4D Scalar-Tensor EFT: RG Flow

① Scalar-Tensor Model: Unknown $F(k,\varphi)$ and $V(k,\varphi)$

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}\left(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)}$$

$$\Downarrow$$

② **RG flow** of F and V given by **Non-linear PDEs of second order** [4]

$$\Downarrow$$

$$k\,V^{(1,0)}(k,\varphi) = \varphi V^{(0,1)}(k,\varphi) - 4V(k,\varphi) + \frac{1}{16\pi^2} + \frac{3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)}{32\pi^2\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\left(V^{(0,2)}(k,\varphi)+1\right)\right)}$$

$$k\,F^{(1,0)}(k,\varphi) = \varphi F^{(0,1)}(k,\varphi) - 2F(k,\varphi) + \frac{37}{384\pi^2}$$
$$+\frac{F(k,\varphi)\left(\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\right)\left(-3F^{(0,2)}(k,\varphi) + 3V^{(0,2)}(k,\varphi) + 1\right) + 2F(k,\varphi)V^{(0,2)}(k,\varphi)^2\right)}{96\pi^2\left(3F^{(0,1)}(k,\varphi)^2 + F(k,\varphi)\left(V^{(0,2)}(k,\varphi)+1\right)\right)^2}$$

[4] Roberto Percacci, Gian Paolo Vacca. arXiv:1501.00888

4D Scalar-Tensor EFT: RG Flow

① Scalar-Tensor Model: Unknown $F(k,\varphi)$ and $V(k,\varphi)$

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}\left(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)}$$

$$\Downarrow$$

② **RG flow** of F and V given by **Non-linear PDEs of second order**

$$\Downarrow$$

③ **Shift-symmetric** ($\varphi \to \varphi + c$), **UV-fixed point** [5]

$$\boxed{\lim_{k\to\infty}\{F(k,\varphi), V(k,\varphi)\} = \{F_*, V_*\} = \{\frac{41}{768\pi^2}, \frac{3}{128\pi^2}\}}$$

[5] Roberto Percacci, Gian Paolo Vacca. arXiv:1501.00888

4D Scalar-Tensor EFT: RG Flow

① Scalar-Tensor Model: Unknown $F(k,\varphi)$ and $V(k,\varphi)$

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}\left(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)}$$

$$\Downarrow$$

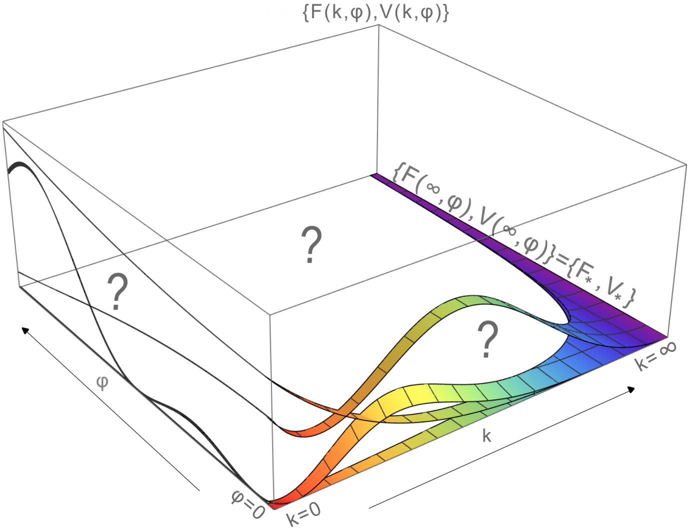
④ **3 Relevant directions:**[6] $F \simeq F_* + \delta F\left(\frac{k0}{k}\right)^\theta$ and $V \simeq V_* + \delta V\left(\frac{k0}{k}\right)^\theta$

$$(\delta F,\delta V)_1 = (0,1) \quad (\delta F,\delta V)_2 = (1,0) \quad (\delta F,\delta V)_3 = (-1+32\pi^2\varphi^2, 0)$$

$\{\theta_1 = 4,\ \theta_2 = 2,\ \theta_3 = 0\}$ → *Higher order analysis* → *All relevant*!

[6] Roberto Percacci, Gian Paolo Vacca. arXiv:1501.00888

4D Scalar-Tensor EFT: Program towards objective



- Boundary condition at $k = \infty$ is settled by the fixed point.
- We have to fix boundary conditions in the φ direction, a priori not determined by the UV fixed point.
- We are interested on the $k = 0$ curves, which are the predictions.

4D Scalar-Tensor EFT: Splitting gravity and scalar field

① Scalar-Tensor Model: Unknown $F(k,\varphi)$ and $V(k,\varphi)$

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi)}$$

$$\Downarrow$$

We separate the constant parts of F and V and define $(\phi^2 \equiv G_k\varphi^2)$

$$\boxed{\Gamma(k) = \int d^4x\sqrt{g}(-\frac{1+f(k,\phi^2)}{16\pi G_k}R + \frac{2\lambda_k + v(k,\phi^2)}{(16\pi G_k)^2} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi)}$$

$$\Downarrow$$

③ UV-fixed point is now expressed as

$$\boxed{\lim_{k\to\infty}\{G_k,\lambda_k\} = \{\frac{48\pi}{41}, \frac{6912\pi^2}{1681}\} \qquad \lim_{k\to\infty} f(k,\phi^2) = \lim_{k\to\infty} v(k,\phi^2) = 0}$$

4D Scalar-Tensor EFT: Boundary conditions in field direction

One can expand the flow equations near $\phi = 0$, assuming

$$f(k,\phi^2) \simeq f_1(k)\phi^2 \quad v(k,\phi^2) \simeq v_1(k)\phi^2$$

Furthermore, if one assumes $f_1(k), v_1(k) << 1$, one obtains

$$12\sqrt{G_k}\,\beta_G f_1^2 + \left(96\pi^2 - 82\pi G_k\right) f_1' = 0 \qquad 82\sqrt{G_k}\,\beta_G\, v_1 + \left(48\pi - 41 G_k\right) v_1' = 0$$

and the solutions are (m_0^v, m_0^f are free parameters)

$$v^{(0,1)}(k,0) = v_1(k) = \frac{41\left(\frac{48\pi}{41} - G_k\right) m_0^v}{48\pi} \qquad \lim_{k\to 0} v_1(k) = m_0^v \qquad \lim_{k\to\infty} v_1(k) = 0$$

$$f^{(0,1)}(k,0) = f_1(k) = \frac{m_0^f}{1 - \frac{3}{41\pi}\, m_0^f \log\left(\frac{48\pi - 41G_k}{48\pi}\right)} \qquad \lim_{k\to 0} f_1(k) = m_0^f \qquad \lim_{k\to\infty} f_1(k) = 0$$

4D Scalar-Tensor EFT: RG Flow

Scalar-Tensor Model: Unknown $f(k,\phi^2)$ and $v(k,\phi^2)$ $(\phi^2 \equiv G_k \varphi^2)$

$$\boxed{\Gamma(k) = \int d^4x \sqrt{g}\left(-\frac{1+f(k,\phi^2)}{16\pi G_k} R + \frac{2\lambda_k + v(k,\phi^2)}{(16\pi G_k)^2} + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi\right)}$$

$$\Downarrow$$

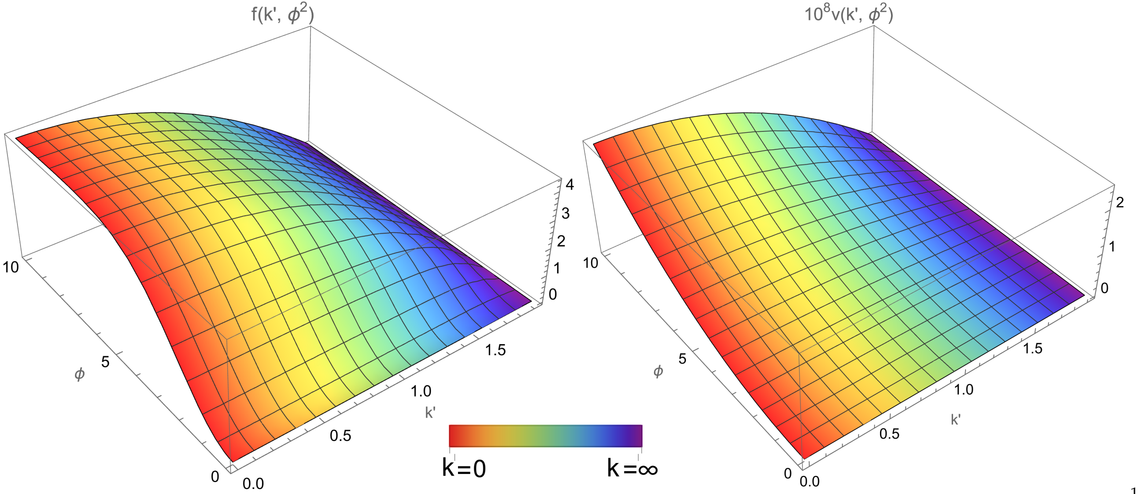
④ The RG flow has 4 **degrees of freedom**, but 2 are fixed by observations!

$$\tilde{G}_0 \simeq 6.71 \times 10^{-39} GeV^{-2}, \quad \lambda_0 \simeq 10^{-120}, \quad \boxed{m_0^f \equiv \frac{\partial^2 f}{\partial \phi^2}\Big|_{\phi=0,k=0} \qquad m_0^v \equiv \frac{\partial^2 v}{\partial \phi^2}\Big|_{\phi=0,k=0}}$$

Only 2 free parameters: m_0^f, m_0^v

4D Scalar-Tensor EFT: Numerical Solutions

$$m_0^f = 0.42 \qquad m_0^v = 2.75 \times 10^{-10}$$



Emergence of Inflation from the UV

By means of a Weyl transformation ($g \to g_E$), one goes to the Einstein frame

$$\Gamma_k = \int d^4x \sqrt{g_E}\left(-\frac{R_E}{16\pi G_k} + \frac{V_{eff}(k,\phi(\sigma))}{(16\pi G_k)^2} + \frac{\partial_\mu\sigma\partial^\mu\sigma}{2\,G_k}\right)$$

Where the effective potential turns out to be

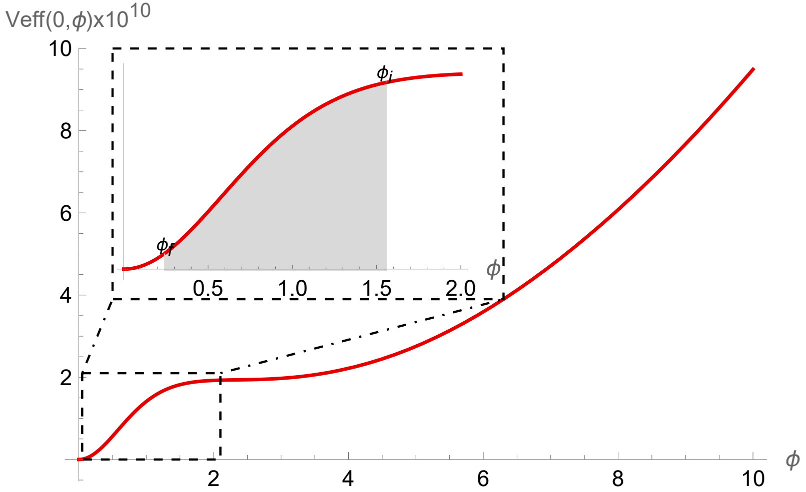
$$\boxed{V_{eff}(k,\phi(\sigma)) \doteq \frac{2\lambda_k + v(k,\phi^2(\sigma))}{(1+f(k,\phi^2(\sigma)))^2}}$$

with the UV and IR limits

$$\lim_{k\to\infty} V_{eff}(k,\phi(\sigma)) = 2\lambda_* \quad \Big| \quad \lim_{k\to 0} V_{eff}(k,\phi(\sigma)) = \frac{2\lambda_0 + v(0,\phi^2(\sigma))}{(1+f(0,\phi^2(\sigma)))^2}$$

Emergence of Inflation from the UV

$$\lim_{k\to 0} V_{eff}(k, \phi(\sigma))$$



Slow-Roll Inflation

$\phi_i = 1.55$

$\phi_f = 0.25$

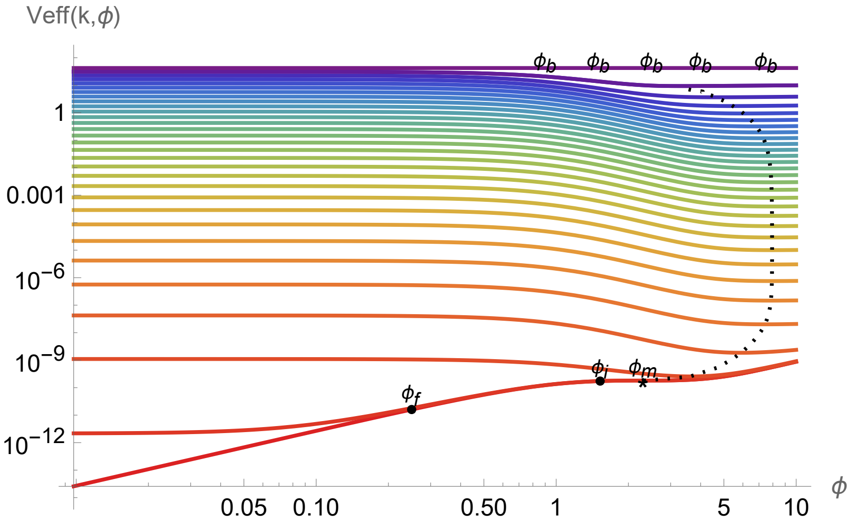
$n_s \simeq 0.965$

$r \simeq 0.005$

$N_{ef} \simeq 66$

$A_s \simeq 2.06 * 10^{-9}$

Emergence of Inflation from the UV



Universe starts near UV-fixed point with arbitrary initial value of ϕ (shift-symmetry)

$\Downarrow$

Adiabatical evolution gives initial condition of inflaton!

Take home messages

We studied the Renormalization Group Flow of scalar-tensor theories and found a UV-Fixed point (Asymptotic Safety).

$\Downarrow$

We connected the UV with the IR by solving the RG flow equations and obtained non trivial emergent potentials.

$\Downarrow$

The emergent potentials could give rise to an inflationary period fitting current observations.

$\Downarrow$

RGF improvement can potentially explain the initial conditions of inflation.

☺ Thank you for your attention ☺

Future?

- Explore expansions of the truncation? Maybe a non trivial kinetic term...
- Explore equations without approximations?
- Explore gauge and regulator dependence?
- Any ideas from you?

☺ Thank you for your attention ☺

Appendix 0: RG Flow Equations

In order to determine if the superficially marginal direction is relevant we expand around

$$F(k,\varphi) \simeq F_* - \delta F0(k) + 32\pi\delta F1(k)\varphi^2 \quad V(k,\varphi) \simeq V_*\,,$$

and obtain the flow equations for the couplings $\delta F0(k)$, and $\delta F1(k)$

$$k\,\frac{\partial\,\delta F1(k)}{\partial k} = -\frac{3072\pi^2\delta F1(k)^2}{41} \qquad k\,\frac{\partial\,\delta F0(k)}{\partial k} = -2(\delta F0(k) - \delta F1(k)),$$

with solutions

$$\delta F1(k) = \frac{41}{3072\pi^2\log\left(\frac{k}{\mathrm{k1}}\right)} \qquad \delta F0(k) = \frac{\mathrm{k0}^2}{k^2} - \frac{41\mathrm{k1}^2\mathrm{Ei}\left(2\log\left(\frac{k}{\mathrm{k1}}\right)\right)}{1536\pi^2k^2},$$

where $k0$ and $k1$ are finite positive real numbers, Ei is the exponential integral, and because we are expanding near the NGFP, we assumed $k >> k0,\, k1$. Therefore $\lim_{k\to\infty}\{\delta F0(k),\ \delta F1(k)\} = \{0,\ 0\}$, thus making the superficially marginal perturbation a relevant one.

Appendix 1: Weyl Transformation

The Weyl transformation to go from the Jordan frame to the Einstein frame is

$$g^E_{\mu\nu} \equiv (1 + f(k, \phi^2))g_{\mu\nu}$$

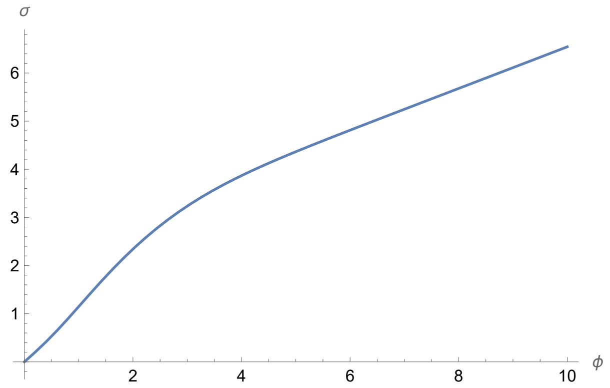
$$\Downarrow$$

For slow-roll inflation this gives the relations at $k = 0$

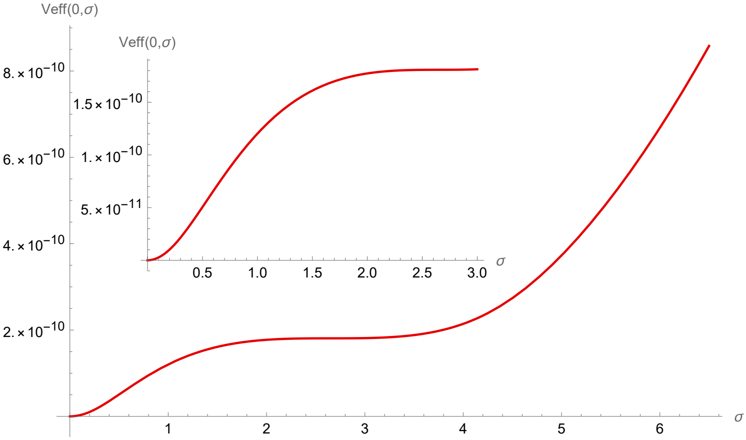
$$\phi_i = 1.55 \rightarrow \sigma_i \simeq 1.8$$

$$\phi_f = 0.25 \rightarrow \sigma_f \simeq 0.26$$

$$\left(\frac{\partial \sigma}{\partial \phi}\right)^2 = \frac{1}{(1 + f(k, \phi^2))} + \frac{3\,(f'(k, \phi^2))^2}{(1 + f(k, \phi^2))^2}$$



Appendix 1: Weyl Transformation



One can of course, compute the potential as a function of the new field σ. The overall shape is similar to the shape as a function of ϕ.

$$\sigma_i \simeq 1.8$$

$$\sigma_f \simeq 0.26$$

$$n_s \simeq 0.965$$

$$r \simeq 0.005$$

$$N_{ef} \simeq 66$$

$$A_s \simeq 2.06 * 10^{-9}$$

Appendix 2: Slow-roll observables

$$(\frac{\partial \sigma}{\partial \phi})^2 = \frac{1}{(1+f(k,\phi^2))} + \frac{3\,(f'(k,\phi^2))^2}{(1+f(k,\phi^2))^2}$$

Einstein Frame	Jordan Frame
$\epsilon = \frac{1}{8\pi}\frac{1}{2}(\frac{V'_{eff}(\sigma)}{V_{eff}(\sigma)})^2 \quad \eta = \frac{1}{8\pi}(\frac{V_{eff}(\sigma)''}{V_{eff}(\sigma)})$	$\epsilon = \frac{1}{8\pi}\frac{1}{2}(\frac{V'_{eff}(\phi)}{V_{eff}(\phi)})^2(\frac{\partial \sigma}{\partial \phi})^{-2}$
	$\eta = \frac{1}{8\pi}((\frac{V_{eff}(\phi)''}{V_{eff}(\phi)}) - (\frac{V'_{eff}(\phi)}{V_{eff}(\phi)})\frac{\frac{\partial^2 \sigma}{\partial \phi^2}}{\frac{\partial \sigma}{\partial \phi}})(\frac{\partial \sigma}{\partial \phi})^{-2}$
$n_s = 1 - 6\,\epsilon + 2\,\eta \quad r = 16\,\epsilon$	$n_s = 1 - 6\,\epsilon + 2\,\eta \quad r = 16\,\epsilon$
$A_s = \frac{1}{4}\frac{V_{eff}}{24\pi^2\epsilon} \quad N_{ef} = 8\pi\int_{\sigma_i}^{\sigma_f}\frac{d\,\sigma}{\sqrt{2\,\epsilon}}$	$A_s = \frac{1}{4}\frac{V_{eff}}{24\pi^2\epsilon} \quad N_{ef} = 8\pi\int_{\phi_i}^{\phi_f}\frac{d\,\phi}{\sqrt{2\,\epsilon}}\frac{\partial \sigma}{\partial \phi}$

Appendix 3: Dimensionless variables

The determination of the RG flow and the existence of fixed points is usually done for dimensionless variables. In our case, we start with dimensionful ($\tilde{}$)

$$\Gamma(k) = \int d^4\tilde{x}\sqrt{g}\left(-\tilde{F}(k,\tilde{\varphi})\tilde{R} + \tilde{V}(k,\tilde{\varphi}) + \frac{1}{2}\partial_\mu\tilde{\varphi}\partial^\mu\tilde{\varphi}\right)$$

and end up with

$$\Gamma(k) = \int d^4x\sqrt{g}\left(-F(k,\varphi)R + V(k,\varphi) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right),$$

where $F \equiv \frac{\tilde{F}}{k^2}$, $V \equiv \frac{\tilde{V}}{k^4}$, $\varphi \equiv \frac{\tilde{\varphi}}{k}$, $R \equiv \frac{\tilde{R}}{k^2}$ and $x \equiv \tilde{x}\,k$.

More in detail, if one has individual couplings instead of functionals, like the newton coupling, one uses $G_k \equiv \tilde{G}_k\,k^2$.

Notice that since $\lim_{k\to\infty} G_k = G_*$ then $\lim_{k\to\infty} \tilde{G}_k = 0$.

Appendix 4: Fixing Constant Dimensionless field on the flow

Usually one writes the flow equations in terms of the dimensionless variables φ defined as $\tilde{\varphi} = \frac{\varphi}{k^{(d-2)/2}}$, and keeping them constant as functions of k. For this, one usually defines, for example $\tilde{F}(k, \tilde{\varphi}) = k^2 F(t, \varphi)$. In this work we used other variables defined as $\psi = \phi^2 = G\varphi^2$, and we kept them constant as functions of k. This amounts to the transformations

$$F(t, G\varphi^2) \doteq f(t, \psi)$$

$$F^{(0,n)}(t, \varphi) \to G^n f^{(0,n)}(t, \psi)$$

$$\frac{\partial F^{(0,n)}(t, \varphi)}{\partial t} \to G^n\left(\frac{n\beta_G}{G} f^{(0,n)}(t, \psi) + f^{(1,n)}(t, \psi) + \psi \frac{n\beta_G}{G} f^{(0,n+1)}(t, \psi)\right)$$

Appendix 5: Final Equations S_{UV}

The change of variables realized to compactify the domain is $k' \equiv G(k)$. Furthermore, we also used $\psi \equiv \phi^2$. This amounts to replacing $f(k,\phi^2) \to f(G,\psi)$ and $v(k,\phi^2) \to v(G,\psi)$, and the respective derivatives $kf^{(1,0)}(k,\phi^2) \to f^{(1,0)}(G,\psi)\beta_G$ and $k\,v^{(1,0)}(k,\phi^2) \to v^{(1,0)}(G,\psi)\beta_G$, and the obvious chain rule for ψ. This allows us to solve the equations in the domain $G \in [0, G_*]$ instead of $k \in [0,\infty]$. The resulting equations are

$$
\begin{aligned}
0 = {} & \frac{256\pi G^7 \left(4\pi(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right) - v^{(0,1)}(G,0)\left(3\psi f^{(0,1)}(G,\psi)^2 + 4\pi(f(G,\psi)+1)\right)\right)}{\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)} \\
& - \frac{\left(24576\pi^3G^7 f^{(0,1)}(G,0) - G\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)\left(2v(G,\psi)-\psi v^{(0,1)}(G,\psi)\right)}{24\pi\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
& - \frac{G^2\left(\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-24576\pi^3G^6f^{(0,1)}(G,0)\right)v^{(1,0)}(G,\psi)}{48\pi\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
& + G\left(-2\psi v^{(0,1)}(G,\psi)+4v(G,\psi)\right)
\end{aligned}
$$

Appendix 5: Final Equations S_{UV}

$$
\begin{aligned}
0 = &-\frac{2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-49152\pi^3G^6f^{(0,1)}(G,0)}{2G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
&-\frac{\psi\left(24576\pi^3G^6f^{(0,1)}(G,0)-\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)f^{(0,1)}(G,\psi)}{G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
&-\frac{\left(\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)-24576\pi^3G^6f^{(0,1)}(G,0)\right)f(G,\psi)}{G^2\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
&-\frac{\left(24576\pi^3G^6f^{(0,1)}(G,0)-\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)\left(\left(45G^2-48\pi\right)v^{(0,1)}(G,0)+128\pi^2\left(41G^2-48\pi\right)G^2\right)\right)f^{(1,0)}(G,\psi)}{2G\left(v^{(0,1)}(G,0)+128\pi^2G^2\right)^2} \\
&+\frac{8(f(G,\psi)+1)\left(256\pi^2G^4\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)\left(-3f^{(0,1)}(G,\psi)-6\psi f^{(0,2)}(G,\psi)+8\pi\right)\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2} \\
&+\frac{8(f(G,\psi)+1)\left(48\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2}+\frac{48\pi\psi(f^{(0,1)}-f(G,\psi)-1)(G,\psi)}{G^2} \\
&+\frac{8(f(G,\psi)+1)\left((f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)^2\right)}{\left(32\pi G^2\left(3\psi f^{(0,1)}(G,\psi)^2+4\pi(f(G,\psi)+1)\right)+(f(G,\psi)+1)\left(v^{(0,1)}(G,\psi)+2\psi v^{(0,2)}(G,\psi)\right)\right)^2}+37
\end{aligned}
$$

Appendix 6: Allowed boundary conditions

One can expand the flow equations near $\phi = 0$, by assuming $f(G, \phi^2) = f_1(G)\phi^2$ and $v(G, \phi^2) = v_1(G)\phi^2$ and expanding up to order ϕ^2. Furthermore, if one assumes $f_1(G), v_1(G) << 1$, one can expand to the lowest non-trivial order in f_1 and v_1. The resulting equations are

$$82G\, v_1(G) + \left(48\pi - 41G^2\right) v_1'(G) = 0$$

$$12G f_1(G)^2 + \left(96\pi^2 - 82\pi G^2\right) f_1'(G) = 0$$

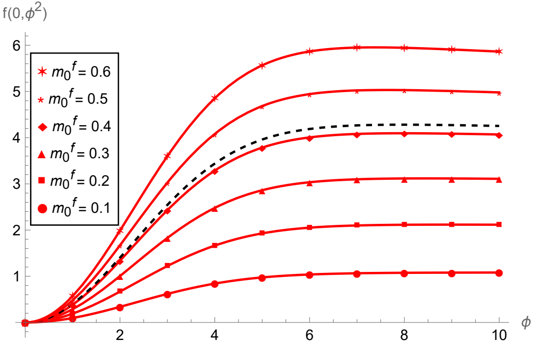
and the solutions are (m_0^v, m_0^f are free parameters)

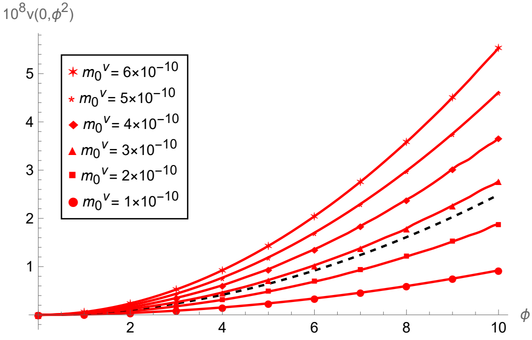
$$v^{(0,1)}(G,0) = v_1(G) = \frac{41\left(\frac{48\pi}{41} - G^2\right) m_0^v}{48\pi} \qquad \lim_{G\to 0} v_1(G) = m_0^v \qquad \lim_{G\to G_*} v_1(G) = 0$$

$$f^{(0,1)}(G,0) = f_1(G) = \frac{m_0^f}{1 - \frac{3}{41\pi} m_0^f \log\left(\frac{48\pi - 41G^2}{48\pi}\right)} \qquad \lim_{G\to 0} f_1(G) = m_0^f \qquad \lim_{G\to G_*} f_1(G) = 0$$

Appendix 7: Different choices of free parameters m_0^v and m_0^f

Different choices for the parameters m_0^v, m_0^f lead to different results of the emergent effective potentials. This is somewhat similar to the swampland program in string theory, where only some parameters are compatible with observations.





Appendix 8: Scaling of interactions and k limits

The construction of the S_{UV} is subject to the knowledge of the limits of the interactions as functions of the scale k. In the case of a negative mass dimension coupling $\tilde{G} = \frac{G}{k^n}$ (like Newton's constant)

$$\lim_{k->0} \tilde{G} = \tilde{G}_0 = \lim_{k->0} \frac{G}{k^n} \quad \rightarrow \quad \lim_{k->0} G = 0$$

In this case, one can use the coupling G to turn all the other relevant interactions (u^α) into negative mass dimension, and all the irrelevant interactions (v^ν) into dimensionless. In this case, the S_{UV} will be functions $v(u)$, with domain $[0, u_*]$, where $v(u_*) = v_*$ in the UV, and $v(0) = v$ is the effective coupling in the IR. An example of how to make a coupling dimensionless, can be the cosmological constant

$$\frac{2\tilde{\Lambda}}{16\pi\tilde{G}} \rightarrow \frac{2\lambda}{(16\pi\tilde{G})^2} \quad (\lambda \equiv 16\pi\tilde{G}\tilde{\Lambda})$$

Appendix 8: Scaling of interactions and k limits

The same thing happens with the fields. For example, a scalar field $\tilde{\varphi} = \frac{\varphi}{k^{(d-2)/2}}$. Since we want to work with variables that finite in the limit of $k \to 0$, to be able to do numerical calculations, we can work at constant $\psi \doteq \tilde{G}\tilde{\varphi}^2 = G\varphi^2$ where

$$\lim_{k->0} \tilde{G}\tilde{\varphi}^2 = \lim_{k->0} G\varphi^2 = \tilde{G}_0\tilde{\varphi}^2 = \psi < \infty$$

also

$$\lim_{k->\infty} \tilde{G}\tilde{\varphi}^2 = \lim_{k->\infty} G\varphi^2 = G_*\varphi^2 = \psi < \infty$$

working with these type of field variables allows one to map the UV and the IR without needing to to infinite values of the field, as one would have to do when working with constant φ defined as $\tilde{\varphi} = \frac{\varphi}{k^{(d-2)/2}}$. This is because the only well defined variable between those 2 in the IR is $\tilde{\varphi}$, and for finite $\tilde{\varphi}$, one must study $\varphi \to \infty$.