Asymptotically safe - canonical

quantum gravity junction

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TOC:

- Introduction and model
- Canonical q'ion and Schwinger functions for Lorentzian QG
- ASQG renormalisation
- Conclusion

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Motivation

- It is widely believed that QG must be formulated non-perturbatively
- ASQG and CQG are such non-pert. programmes
- However, apparently profoundly different:

	signature	background methods	truncations
ASQG	mostly Euclidian	essential	widely used
CQG	Lorentzian	absent	so far dispensable

- has prevented interaction btw. research fields to date
- Q: Are these differences truly unsurmountable?
- A: Not really, with proper adjustments understood

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- To explore possible ASQG - CQG interface: formulate CQG in language of ASQG
- Reminder: CQG IS QFT of QG [DeWitt,Dirac,Wheeler,..] e.g. LQG corr. to specific state
- General framework in [TT, Ferrero & TT; 24]
- This talk: Concrete implementation in crystal clear model
- Strategy: reduced phase space (r.p.s.) formulation of Lorentzian CQG: gauge invariance manifest, will never talk about non-observables
- Construct r.p.s. path integral (PI): Euclidian QFT formulation
- Integrating out momenta: necessary measure adjustments absorbed by canonical transformation in CQG formulation
- Result: Euclidian QFT of Lorentzian QG as highly non-linear $\sigma-$model
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Concrete model: Gaussian dust

- Gaussian dust action [Kuchar, Torre 90's] coupled to $D+1$ dim Lorentzian GR $(\mu, \nu = 0, .., D)$

$$L_{GD} = -|\det(g)|^{1/2} \, [\frac{\rho}{2} \, (g^{\mu\nu} \; T_{,\mu} \; T_{,\nu} + 1) + g^{\mu\nu} \; T_{,\mu} \; (W_j \; S^j_{,\nu})]$$

- 2 x (1+D) minimally coupled scalar fields $(T, \ \rho), \ (S^j, \ W_j), \ j = 1, .., D$
- perfectly generally covariant
- classical physics (Euler-Lagrange eqns.):
  - $U_\mu = \nabla_\mu T$: unit timelike geodesic co-tangent $\perp$ to S^j =const. lines
  - pressureless $p = D^{-1}[U_\mu \ U_\nu + g_{\mu\nu}] \ T^{\mu\nu} = 0$
  - interpretation: collision free, synchronised geodesic observer congruence labelled by S^j, proper time T coupled to GR (backreaction)
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- classical Hamiltonian formulation $M \cong \mathbb{R} \times \sigma$ [Giesel, TT 10's]
- k = D(D+1)/2 + 3 (D+1) canonical pairs ($a, b, c, .. = 1, .., D$):
  $(q_{ab}, p^{ab}),\ (N^\mu, \pi_\mu),\ (T, I),\ (S^j, I_j),\ (\rho, Z),\ (W_j, Z^j)$
- Legendre transf. sing.: 2 x (D+1) + (D-1) velocities u^μ, v, v_j, w^A of $N^\mu, \rho, W_j, S^A;\ A = 1, ., ., D-1$ not solvable for
- Dirac's constraint analysis:
  - 2 x (D+1) + (D-1) primary constraints
    $\pi_\mu = Z = Z^j = \zeta_A = W_D I_A - W_A I_D = 0$
  - primary Hamiltonian

    $$h = u^\mu\ \pi_\mu + v\ Z + v_j\ Z^j + w^A\ \zeta_A + N^\mu\ c_\mu$$

  - (D+1) + 2 secondary constraints $c_\mu = \zeta = \zeta_D = 0$; 2 x D velocities $v = v^*, v_j = v_j^*, w^A = w_*^A$ fixed
  - f = 2 x (D+1) first class constr.: π_μ, c_μ,
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    - f = 2 x (D+1) first class constr.: π_μ, c_μ, s= 2 x (D+1) second class constr. Z, Z_j, ζ, ζ_j
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- k = D(D+1)/2 + 3 (D+1) canonical pairs ($a, b, c, .. = 1, .., D$): $(q_{ab}, p^{ab}),\ (N^\mu, \pi_\mu),\ (T, I),\ (S^j, I_j),\ (\rho, Z),\ (W_j, Z^j)$
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Canonical quantisation

- construct 1-para family of conjugate canonical pairs (motivation: later)

$$Q^r_{ab} = [\det(q)]^r \; q_{ab}, \; P^{ab}_r = [\det(q)]^{-r} \; [p^{ab} - \frac{r}{1+rD} \, q^{ab} \, q_{cd} \, p^{cd}]$$

$$\{P^{ab}_r(x), \; Q^r_{cd}(y)\} = \kappa \; \delta^a_{(c} \; \delta^b_{d)} \; \delta(x,y)$$

- New aspect: (Q, P) carry density weights $(2r, 1-2r)$
- Let $\mathfrak{A}_r$: time zero Weyl algebra generated by Weyl el. $W_r(f,g) = e^{i \int_\sigma d^Dx \, [f^{ab} \, Q^r_{ab} + g_{ab} \, P^{ab}_r]}$
- Task 1: Find states (i.e. pos., lin., normalised functionals) $\omega : \mathfrak{A}_r : \to \mathbb{C}$
- Proposition [Gel'fand, Naimark, Segal] ω is equivalent to GNS data $(\rho, \mathcal{H}, \Omega)$ via $\omega(\mathfrak{a}) = < \Omega, \; \rho(\mathfrak{a}) \; \Omega >_{\mathcal{H}}$
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- Proposition When integrating out the momenta, there is a non-trivial measure Jacobean coming from the DeWitt-metric ($r = 0$)

$$G_{abcd} = [\det(q)]^{-1/2}[q_{a(c}\, q_{d)b} - \frac{1}{D-1} q_{ab}\, q_{cd}]$$

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- Except for Gibbons-Hawking and state dependent boundary term, integrand equals Euclidian signature metric EH action in synchronous gauge ...
- ... despite the fact that Hamiltonian for Lorentzian signature GR
- No contradiction: just Wick rotat., formally $N_L = 1 \to N_E = i$ [Niedermaier et al]
- No complex valued metrics arise because H not explicitly time dependent.
- For $r = r_D$, formal Lebesgue measure, else measure correction
- Cf. ASQG field redefinitions works [Baldazzi, Falls, Ohta, Percacci, Pereira, Zinati]
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$$\int d^{D+1}X \, [\det(Q)]^{\frac{1}{2(1+rD)}} \, \{\frac{1}{2} \, K^{abcd}(r) \, \dot{Q}_{ab} \dot{Q}_{cd} - [R([\det(Q)]^{-\frac{r}{1+rD}} Q) - 2\Lambda]\},$$

$$K^{abcd}(r) = \frac{1}{2}[Q^{a(c} \, Q^{d)b} - u' \, Q^{ab} \, Q^{cd}], \; u' = \frac{u}{uD-1}, \; u = \frac{1 + 2r + r^2 \, D}{D-1}$$

Generating functional of Schwinger functions

Discussion:

- Except for Gibbons-Hawking and state dependent boundary term, integrand equals Euclidian signature metric EH action in synchronous gauge ...
- ... despite the fact that Hamiltonian for Lorentzian signature GR
- No contradiction: just Wick rotat., formally $N_L = 1 \to N_E = i$ [Niedermaier et al]
- No complex valued metrics arise because H not explicitly time dependent.
- For $r = r_D$, formal Lebesgue measure, else measure correction
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- Standard steps: background field method and cut-off (Ω dep. not displayed)

$$\bar{Z}_k[F;\bar{Q}] = \int [dH]\; e^{S[\bar{Q}+H]}\; e^{<F,H>}\; e^{-\frac{1}{2}\; R_k(H;\bar{Q})}$$

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$$\bar{\Gamma}_k[\hat{Q},\bar{Q}] = \text{extr}_F\; \{< F,\hat{Q} > - \ln(\bar{Z}_k[F;\bar{Q}])\} - \frac{1}{2}\; R_k(\hat{Q};\bar{Q})$$

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- No gauge fixing, no ghosts: gauge reduction before q'ion, correlation functions of $\hat{Q}$ have immediate physical meaning
- Point of view of CQG:
  - object of physical interest: true effective action (1-PI generating functional) $\Gamma[\hat{Q}] := \bar{\Gamma}_k[\hat{Q}';\bar{Q}]_{\hat{Q}'=0,\bar{Q}=\hat{Q},k=0}$ background independent
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Cut-off kernels 1: Laplacians

- Action, Hamiltonian no longer inv. wrt full $\text{Diff}_{D+1}(\mathbb{R}\times\sigma)$, only wrt subgroup $\text{Diff}_D(\mathbb{R}\times\sigma)$ of time preserving diffeos $\Phi(s,x)=(s,\varphi(x)),\ \varphi\in\text{Diff}_D(\sigma)$.
- This aspect similar to Horava-Lifshitz gravity (HL-GR)
- Classify irreducible tensor fields wrt $\text{Diff}_D(\mathbb{R}\times\sigma)$ by type $S_D(A,B,w)$.
- irreps $T_{D+1}(A,B,w)$ wrt $\text{Diff}_{D+1}(\mathbb{R}\times\sigma)$ decompose into irreps of $\text{Diff}_D(\mathbb{R}\times\sigma)$
- General form of cut-off kernel: $R_k^{abcd}((s,x),(s',x');\bar{Q}):\ S_D(0,2,w)\to S_D(2,0,w),\ w=2r$
- Want to import heat kernel techniques developed for T_{D+1} but how?

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  By corollary: $I_N = \delta_{N,0}, N \geq 0$. Stieltjes moment problem: uniquely $\hat{r}(t) = \delta(t)$.
  By corollary: $I_N = \infty = \frac{1}{|N|!}$ contradiction (reason: Paley-Wiener)
- To be safe & tame sing. convol. t integrals pick $\hat{r}$ smooth, rapid $t = 0, \infty$ decay
- example: $\hat{r}(t) = e^{-[t^2+t^{-2}]}$
- Convol. sing. heat kernel time integrals are of type ($\lambda > 0,\ p > 0\ n \geq m \geq 1$)

$$J_{p,m,n}(\lambda) := \int_{[0,\infty]^n} d^n t\ \prod_{k=1}^m\ \hat{r}(t_k)\ e^{-\lambda \sum_{l=m+1}^n\ t_l}\ (\sum_{k=1}^n t_k)^{-p}$$

- Conv., analyt. fn. of λ, computable Taylor coeff. (generalised Bessel fns.) using Barnes factorising integral identities

$$(s_1+s_2)^{-p} = \int_{-\frac{1}{4}-i\infty}^{-\frac{1}{4}+i\infty} \frac{dz}{2\pi\ i}\ s_1^z\ s_2^{-[p+z]}\ \frac{\Gamma(z+p)\Gamma(-z)}{\Gamma(p)}$$

Cut-off kernels 3: tensor structure

- When computing $\bar{\Gamma}_k^{(2)}(\hat{Q},\bar{Q})$ for the Wetterich eqn. a new effect arises when $r \neq 0$, structurally

$$< H,\ [\bar{\Gamma}_k^{(2)}(\hat{Q},\bar{Q})]_{\hat{Q}=0} \cdot H >_D = < H,\ \{K_1(r)\cdot[-\partial_s^2] + K_2(r)\cdot[\overline{\Delta}_D + \partial_s^2 + 2\Lambda_k] + U_k\}\cdot H >_D$$

- U_k: non-minimal terms
- Time and space der. have different coeff.: $K_1(r) - K_2(r) \propto r \neq 0$ unless $D = 4$
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- ... although conceptual setup quite different: only true d.o.f. PI (no ghosts), different treatment of time derivatives, unitary vs STT gauge, $r-$ dependent density weight
- important to CQG: existence of true E.A. Γ i.e. finite dimensionful couplings as $k \to 0$, of course depending on trajectory (relevant couplings)

- Beta fns: non-trivial r-dependence, polynomial in g, analyt. in λ
- UV NGFP $\lambda_* = 1.92,\ g_* = 57.41$, IR GFP $\lambda_* = g_* = 0$
- crit. exp. $(\lambda - \lambda^*, g - g_* \propto [\frac{k_0}{k}]^{\theta_{1/2}})$: $(\theta_1, \theta_2) = (8.01, 2.13)$ (NGFP) (2, -2) (GFP)
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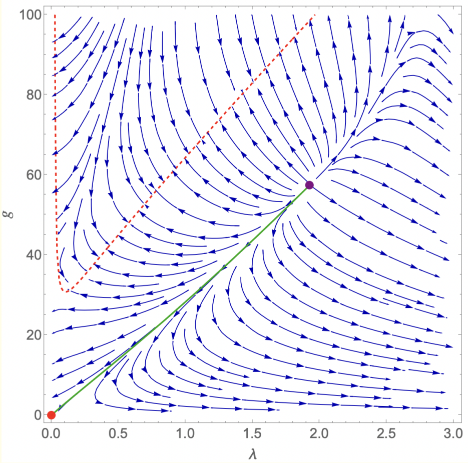
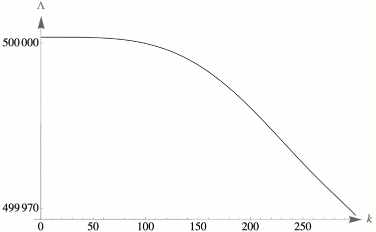


Figure: Flow diagramme in $\lambda - g$ plane for $r_3 = -\frac{1}{12}$, $D = 3$, trajectories point to decreasing k, all originate from UV NGFP (purple dot). Red dashed line: "curtain" (pole line of beta functions, flow unreliable beyond). Green line: separatrix connecting UV NGFP and IR GFP (red dot).



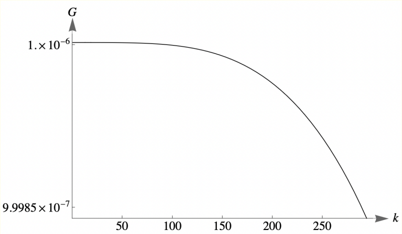


Figure: Small k regime of the dimensionful cosmological constant and Newton's constant. Both couplings reach a finite value when $k \to 0$. This value depends on the initial conditions.

Conclusion

- ASQG and CQG methods in harmonic interaction in concrete matter model with transparent physics
- Lorentzian signature Hamiltonian and Euclidian signature action coexist without contradiction
- Relational formalism leads to different treatment of gauge invariance
- First principle derivation of PI leads to measure Jacobeans which necessarily have non-trivial influence on flow and truncation spaces
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